PMATH 011 – Basic Algebra

INST: i. Answer Question 1 and any other 3 (Three) Questions ii. Show all the working iii. Start every question on a new page

Question 1. (25 marks)

(a) Use the properties of exponential to Simplify the following expressions

(i)
$$x^2 \cdot x^{-6}$$

(ii) $\frac{4^6}{4^2} \times 4^3$
(iii) $(2^2 \cdot x^0)$
(iv) $[(x^2 y^{-3})(xy^3)]^0$
(v) $(\frac{6^4}{4^{-2}}) \cdot (\frac{4^6}{6^{-3}})$

(b) Solve for x in the following expressions

i.
$$3^{2x} = 3^4$$

ii.
$$2^{x+1} = 2^{-4}$$

iii.
$$4^{x+1} = 64$$

(c) solve for x in the following equations;

(*i*)
$$5x - 2 + x = 9 + 3x + 10$$

$$(ii) \quad \sqrt{\left(\frac{4x^2-1}{2x+1}\right)} = 4$$

$$(iii) \quad 3x^2 - 9x - 30 = 0$$

$$(iv) \quad x^2 = 11x - 24$$

- (v) |3x + 5| = 10
- (d) Given $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$. Evaluate the following logarithms in base 10; i) $\log_{10} 6 \div \log_{10} 3$ ii) $\log_{10} 18$ iii) $\log_{10} 3^5$ iv) $\log_3 2$
 - (e) Solve the following logarithmic equations i. $\log_2(x+2) + \log_2 5 = 3$
 - *ii.* $\log_3(x+3) \log_3(x-1) = 1$
 - (f) Use the properties of logarithms to simplify the following;
 - (i) $\log_3 27 + \log_3 9 \log_3 81$
 - (*ii*) $\log_2 8 \log_2 16$
 - (*iii*) $\log_4(64)^{\frac{1}{3}}$

Question 2. (15)

- (a) The sum of two numbers is 20. If 4 is added to the larger number, the result is 3 times the smaller number. Find the numbers.
- (b) The perimeter of the rectangle shown below is 24 cm. what is the value of x?



(c) Use a calculator to evaluate the following;

i)
$${}^{8}C_{6}$$
 ii) ${}^{6}P_{6}$ iii) $\frac{{}^{5}P_{3}}{{}^{8}C_{5}}$

(d) How many possible distinguishable permutations are there in the letters of the words;

i) INTERMEDIATE ii) CHAPTER iii) STATEMENTS

(e) Using Binomial Theorem, write the 5th and the 10th terms in the following expansion;

 $(x-2)^{12}$

Question 3. (15 marks)

(a) find the equation of a line that passes through the following pair of points (4,1) and (2,-3)

(b) find the equation of a line that has the following properties;

- i) passes thro' (2,7) and is perpendicular to the line y = x + 1
- ii) passes thro' (2,7) and is parallel to the line y = 4x + 7
- (c) Given $f(x) = x^2 1$ and g(x) = x 1. Find (i) (f+g)(x) (ii) (f-g)(2) (iii) (f.g)(x) (iv) (f/g)(x)(v) f, g(x)
- (d) Given f(x) = 3x + 5, find the inverse, $f^{-1}(x)$.

(e) If the equation of a line is given as; y = 2x - 3. What is the slope and the y-intercept of this line? Sketch this line.

(f) Write the equation of a line that passes through the following points; (2,0) and (-5,2)

(g) Write the equation of a line that has the following properties; Passes thro' (2,6) and is perpendicular to the line described by the equation y = 3 - 2x

Question 4. (15 marks)

(a) Using the binomial expansion formula approximate the following;

- *i.* $(2.02)^9$ use only the first four terms
- *ii.* $(x+2)^{12}$ write the 8th and 13th terms in the expansion

(b) solve the following absolute/ inequality functions;

- *i*. |3x+4| > 10
- *ii.* 4 3x > 10

(c) The sum of three consecutive odd numbers is 45. Find the numbers.

(d) One leg of a right triangle is 6 cm longer than the other leg. The length of the hypotenuse of the triangle is 24 cm. Find the lengths of the two legs.

(e) How many distinguishable permutations can be formed using the letters in the word COMMISSION ?

Question 5. (15 marks)

(a) Perform the indicated operations if possible.

i. $\begin{bmatrix} -1 & 4 \\ 2 & -6 \end{bmatrix} - \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$	ii. $\begin{bmatrix} 4 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 0 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 1 & 3 \\ 5 & 6 & -8 \\ 3 & 0 & 7 \end{bmatrix}$	
iii. $\begin{bmatrix} -1 & 4 \\ 2 & -6 \end{bmatrix} \times \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$	iv. $\begin{bmatrix} -3 & 5\\ 2 & 0\\ 1 & 4 \end{bmatrix} \times \begin{bmatrix} -2 & 1 & 3\\ 5 & 6 & -8 \end{bmatrix}$	-

(b) Find the determinants of the following matrices if they exist;

	Γ11	4		0	2	1]
i.		4	ii.	3	-1	2
	[-3	U		4	0	1

(c) using the matrix method (Cramer's Rule), solve the following system of linear equations;

$$2x - 3y = -10$$

 $3x + 2y = 11$

(d) Using the matrix method, find the following:

- i. The equation of a line that passes thro' the points ; (-1,3) and (3,6).
- ii. Area of a triangle whose vertices have the following co-ordinates (-1,0) (3,8) and (6,-1).
- iii. Check whether these points are collinear; (0,2), (3,4) and (6,1).