KABARAK UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN COMPUTR SCIENCE

COURSE CODE: MATH 113

COURSE TITLE: CALCULUS I

STREAM: Y1S1

DAY: WEDNESDAY

TIME: 9.00 – 11.00 A.M.

DATE: 12/08/2009

INSTRUCTIONS:

Attempt question **ONE** and any other **TWO** questions.

PLEASE TURN OVER

QUESTION ONE (30MKS)

(a) Evaluate the following limits

(i) $\lim\limits_{x\to\infty} \frac{n^2 - 4n + 7}{2n^2 + 4}$ **(3mks)**

(ii) $\lim\limits_{x\to 0} \frac{\sqrt[m]{1+ax} - \sqrt[n]{1+bx}}{x}$ **(5mks)**

(iii) $\lim\limits_{x\to\infty} \frac{(x-3)^{40}(5x+6)^{10}}{(3x^2-6)^{25}}$ **(4mks)**

(iv) $\lim\limits_{x\to\infty} \frac{x^x Sin\ 2x}{3x}$ **(4mks)**

(b) Find $\frac{dy}{dx}$ of the following functions from first principles

(i) $y = 4x^2 + 2x + 2$ **(3mks)**

(ii) $y = \sqrt{x+2}$ **(3mks)**

(c) Let $\lim x_n \to A$ and $\lim y_n \to B$. Then show that

(i) $\lim\limits_{x\to\infty} x_n + y_n = A + B$ **(4mks)**

(ii) $\lim\limits_{x\to\infty} x_n y_n = A \bullet B$ **(4mks)**

QUESTION TWO (20MKS)

(a) Using the ε - δ definition of limits show that $\lim\limits_{x\to 2} x^2 + 2x + 2 = 10$ and evaluate the value of δwhen ε = 0.1 at $x = 5$ **(8mks)**

(b) Find $\frac{dy}{dx}$ of the following functions **(6mks)**

(i) $y = (Sin\,2x)^{\sin x^{\,coxx}}$ **(4mks)**

(ii) y = $\left(x^3 - 2x\right)^3 \cdot (3x+5)^7$ **(6mks)**

(iii) y = $\sqrt[3]{2x^3 + 4x + 2}$ **(2mks)**

(iv) y = $Ln^3\left(x^2 + 4x + 2\right)^2$ **(3mks)**

QUESTION THREE (20MKS)

(a) Find y^1 given $y = \dfrac{e^{ax} + e^{-ax}}{e^{ax} - e^{-ax}}$ **(6mks)**

(b) Find y^{11} given $y = e^{-2x} \sin 3x$ **(5mks)**

(c) Find y^1 and y^{11} given $x^4 + xy^3 + y^3 = 3$ at the point (1, 1) **(8mks)**

QUESTION FOUR (20MKS)

(a) Given that $xy = Sin^{-1} \ln\left|2x^2 + x\right|$ Find $\dfrac{dy}{dx}$ **(5mks)**

(b) Determine and distinguish the stationary points of the curve; $y = x^3 - 6x^2 + 9x + 2$ and hence state the local extrema. **(7mks)**

(c) Evaluate the following explaining every step.

(i) $\lim\limits_{x \to 0} \dfrac{Ln(1+5x)}{4x}$ **(2mks)**

(ii) $\lim\limits_{x \to \infty} \left(\dfrac{3x+1}{3x-2}\right)^{2x}$ **(3mks)**

(iii) $\lim\limits_{x \to 0} \dfrac{1 - \cos 3x}{x^2}$ **(2mks)**

QUESTION FIVE (20MKS)

(a) Prove $\frac{d}{dx}(uv) = u\frac{d(v)}{dx} + V\frac{d(u)}{dx}$ **(8mks)**

(b) Given $y = \frac{\cos x}{x}$, hence or otherwise prove that $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + xy = 0.$ **(7mks)**

(c) Evaluate $\int \frac{x^2}{5x^3+1}\,dx$ **(3mks)**

(d) Determine whether $y = Ae^{ax} + Ba^{-ax}$ is satisfied by $y^{11} - a^2y = 0$ **(2mks)**