KABARAK

 SCIENCE

## COURSE CODE: MATH 113

## COURSE TITLE: CALCULUS I

STREAM: SESSION I
DAY:
TUESDAY
TIME:
DATE:
11/08/2009

## INSTRUCTIONS:

Attempt question ONE and any other TWO questions.

## QUESTION ONE (30MKS)

(a) (i) Find $\operatorname{Lim}_{x \rightarrow 1} \frac{x^{2}-1}{x^{2}-3 x+2}$

> (3mks)
(ii) Find $\operatorname{Lim}_{x \rightarrow 0} \frac{\sin x-\cos 3 x}{x^{2}}$
(3mks)
(b) Use first principles to find the derivative of
(i) $\mathrm{f}(\mathrm{x})=2 \mathrm{x}$
(3mks)
(ii) $\mathrm{f}(\mathrm{x})=\frac{1}{x}$
(3mks)
(c) Show that $\operatorname{Lim}_{x \rightarrow a} x^{2}=a^{2}$
(d) Find the equation of the line which passes through a point $(5,3)$ and is parallel to the tangent to the curve. $y=x^{2}$ at the point $(3,9)$
(e) Show that the polynomial $\mathrm{P}(\mathrm{x})=3 \mathrm{x}^{3}-\mathrm{x}+5$ is continuous at $\mathrm{x}=1$.
(f) Differentiate the following functions:
(i) $y=\left(x^{2}-5\right)\left(x-x^{3}\right)$
(ii) $y=\frac{3 x^{2}-5}{1-x^{3}}$
(iii) $y=\ln \left(5 x^{2}-2 x+1\right)$
(3mks)
(4mks)
(2mks)

## QUESTION TWO (20MKS)

(a) Use the first principles to find the derivative of:

$$
\begin{equation*}
y=\operatorname{Sin} x \tag{8mks}
\end{equation*}
$$

(b) For the function defined by $y=2 x^{3}-15 x^{2}+24 x+19$, find the stationary points and distinguish between them.
( 6 mks )
(c) Differentiate the function. $\mathrm{y}=\frac{\sin x}{\cos x}$
( 6 mks )

## QUESTION THREE (20MKS)

(a) A particle moves along a straight line in such a way that its distance from a fixed point o on the line after A t seconds is S meters, where $\mathrm{S}=\frac{1}{6} t^{4}$. Find;
(i) Its velocity after 3 seconds and after 4 seconds
(ii) Its acceleration after 2 seconds and after 4 seconds
(3mks)
(3mks)
(b) Find the equation of the normal to the curve $y=x^{2}+3 x$ at the point where $x=1$
(c) Find $\frac{d y}{d x}$, given that $\mathrm{y}=\frac{\operatorname{Tan}^{2} x e^{2 x}}{x^{2}}$
(d) Evaluate $\operatorname{Lim}_{x \rightarrow \infty}\left(1+\frac{4}{x}\right)^{2+x}$

QUESTION FOUR (20MKS)
(a) Find $\frac{d y}{d x}$ and $\frac{d^{2} y}{d x^{2}}$ if $x^{2} y+2 y^{3}+4 x=0$ at a point $(1,1)$
(15mks)
(b) Show that the rational function $\mathrm{f}(\mathrm{x})=\frac{x+1}{(x-2)^{2}}$ is continuous at $\mathrm{x}=3$
(5mks)

## QUESTION FIVE (20MKS)

(a) Differentiate the following:
(i) $y=5 x^{3}-4 x^{2}+12 x-8$
(1mks)
(ii) $y=\operatorname{Sin}^{3} 2 x$
(5mks)
(b) Find $\frac{d y}{d x}$ when $\mathrm{x}=1$ of $\mathrm{y}=\frac{u}{u+1}$ and $\mathrm{u}=3 \mathrm{x}^{2}-1$
(c) Find the composite function $f\left(g(x)\right.$ where $f(u)=u^{2}+3 u+1$ and $g(x)=x+1$ (4mks)
(d) Find the equation of the curve given the gradient is $4 \mathrm{x}-2$ at a point $(1,2)$
(2mks)

