KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2007/2008 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

COURSE CODE: MATH 113

COURSE TITLE: **CALCULUS I**

STREAM: Y1S1

DAY: **WEDNESDAY**

TIME: **8.30 – 10.30 A.M**

DATE: **23/04/2008**

INSTRUCTIONS TO CANDIDATES:

1. Answer Question **ONE** and any other **TWO** Questions
2. Show **ALL** your workings.

PLEASE TURN OVER

QUESTION ONE (30 MARKS) COMPULSORY

(a) Give 4 examples of functions used in real life (4 mks)

(b) Find the domain and range of the following functions

(i) $f(x) = \sqrt{4+x}$ (2 mks)

(ii) $f(x) = \log\left(\frac{1}{4-x}\right)$ (3 mks)

(c) Evaluate the following limits

(i) $\lim_{x\to 1} \frac{x^2-1}{x+1}$

(ii) $\lim_{x\to 5} (2x^2 - 3x+4)$ (2 mks)

(iii) if $f(x) = \begin{cases} \sqrt{x-4} & if \; x \geq 4 \\ 8-2x & if \; x < 4 \end{cases}$

Determine whether $\lim_{x\to 4} f(x)$ exists (4 mks)

(d) Find the constant a so that the following function is continuous

$$f(x) = \begin{cases} x^2 & x > 2 \\ ax+1 & x \leq 2 \end{cases}$$

(4 mks)

(e) From first principle (or using limits) find the derivative of

(i) f(x) = cos x (3 mks)

(ii) $f(x) = \left(\frac{1}{2x+2}\right)$ (3 mks)

(f) Differentiate with respect to x the following

(i) $f(x) = x^2 e^{2(x+3)}$ (3 mks)

(ii) $f(t) = \left(\frac{2x}{4+x^2}\right)$ (3 mks)

(iii) $f(t) = \tan^{-1} 2x$ (4 mks)

QUESTION TWO (20 MARKS)

(a) (i) State Rolle's theorem (3 mks)

(ii) State the mean value theorem (3 mks)

(b) Sketch the curve $y = x^2-4x+3$ (6 mks)

(c) Using L hospital rule, calculate

(i) $\lim_{x \to \infty} \frac{In\ x}{\sqrt[3]{x}}$ (3 mks)

(ii) $\lim_{x \to \infty} \frac{\tan\ x - x}{x^3}$ (5 mks)

QUESTION THREE (20 MKS)

(a) Find the derivatives of

(i) $f(x) = e^x \ln(x + 2)$ (3 mks)

(ii) $y = 3x - x^2 + \log_2 x$ (3 mks)

(b) Find $\lim_{x \to 1} \frac{\sqrt{x^2 + 2} - 3}{x - 1}$ (5 mks)

(c) Let $y^2 - 4xy + 4x - 1 = 0$ give the equation of a graph. Find the equation of the tangent line to this graph at the point (1,3) (5 mks)

(d) Evaluate

(i) $\int_0^1 (x+1)^2\ dx$ (2 mks)

(ii) $\int (4x+6)^3 dx$ (2 mks)

QUESTION FOUR (20 MARKS)

(a) Differentiate with to x

(i) $f(x) = \sin(4x+3)$ (3 mks)

(ii) $f(x) = \frac{x^2}{In\ x^2}$ (3 mks)

(b) A lader 10 fit zone rests against a vertical wall. If the bottom of the ladder slides away form the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6ft form the wall. (6 mks)

(c) From first principle (using limits), find the derivative of

$f(x) = \log_e x$ (5 mks)

(d) $f(x) = 4x$ (3 mks)

QUESTION FIVE (20 MARKS)

(a) (i) Find $\frac{dy}{dx}$ if $x^3 + y^3 = 6xy$ (5 mks)

(ii) Find the tangent to the following equation $x^3 + y^3 = 6xy$ at the point (3,3) (2 mks)

(iii) At what points on the curve is the tangent line horizontal? (3 mks)

(b) Given $f(x) = \frac{1}{x}$, find $f^{--}(x)$ (4 mks)

(c) Find the area under the curve $y = x^2 - 3x + 4$ and the x – axis (4 mks)

(d) The profit function of a product is given by $p(x) = (100 - x)\,x - 28x - 54$. Find the maximum profit of the product. (4 mks)