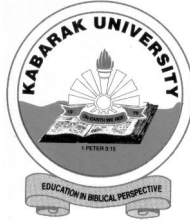


**KABARAK**



**UNIVERSITY**

**UNIVERSITY EXAMINATIONS**

**2007/2008 ACADEMIC YEAR**

**FOR THE DEGREE OF BACHELOR OF SCIENCE IN  
ECONOMICS AND MATHEMATICS**

**COURSE CODE:** MATH 113

**COURSE TITLE:** CALCULUS I

**STREAM:** Y1S1

**DAY:** WEDNESDAY

**TIME:** 8.30 – 10.30 A.M

**DATE:** 23/04/2008

---

**INSTRUCTIONS TO CANDIDATES:**

1. Answer Question **ONE** and any other **TWO** Questions
2. Show **ALL** your workings.

**PLEASE TURN OVER**

**QUESTION ONE (30 MARKS) COMPULSORY**

(a) Give 4 examples of functions used in real life (4 mks)

(b) Find the domain and range of the following functions

(i)  $f(x) = \sqrt{4+x}$  (2 mks)

(ii)  $f(x) = \log\left(\frac{1}{4-x}\right)$  (3 mks)

(c) Evaluate the following limits

(i)  $\lim_{x \rightarrow 1} \frac{x^2-1}{x+1}$

(ii)  $\lim_{x \rightarrow 5} (2x^2 - 3x+4)$  (2 mks)

(iii) if  $f(x) = \begin{cases} \sqrt{x-4} & \text{if } x \geq 4 \\ 8-2x & \text{if } x < 4 \end{cases}$

Determine whether  $\lim_{x \rightarrow 4} f(x)$  exists (4 mks)

(d) Find the constant a so that the following function is continuous

$$f(x) = \begin{cases} x^2 & x > 2 \\ ax+1 & x \leq 2 \end{cases}$$

(4 mks)

(e) From first principle (or using limits) find the derivative of

(i)  $f(x) = \cos x$  (3 mks)

(ii)  $f(x) = \left(\frac{1}{2x+2}\right)$  (3 mks)

(f) Differentiate with respect to x the following

(i)  $f(x) = x^2 e^{2(x+3)}$  (3 mks)

(ii)  $f(t) = \left(\frac{2x}{4+x^2}\right)$  (3 mks)

(iii)  $f(t) = \tan^{-1} 2x$  (4 mks)

**QUESTION TWO (20 MARKS)**

(a) (i) State Rolle's theorem (3 mks)

(ii) State the mean value theorem (3 mks)

(b) Sketch the curve  $y = x^2-4x+3$  (6 mks)

(c) Using L hospital rule, calculate

- (i)  $\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt[3]{x}}$  (3 mks)
- (ii)  $\lim_{x \rightarrow \infty} \frac{\tan x - x}{x^3}$  (5 mks)

**QUESTION THREE (20 MKS)**

- (a) Find the derivatives of
- (i)  $f(x) = e^x \ln(x + 2)$  (3 mks)
- (ii)  $y = 3x - x^2 + \log_2 x$  (3 mks)
- (b) Find  $\lim_{x \rightarrow 1} \frac{\sqrt{x^2 + 2} - 3}{x - 1}$  (5 mks)
- (c) Let  $y^2 - 4xy + 4x - 1 = 0$  give the equation of a graph. Find the equation of the tangent line to this graph at the point (1,3) (5 mks)
- (d) Evaluate
- (i)  $\int_0^1 (x+1)^2 dx$  (2 mks)
- (ii)  $\int (4x+6)^3 dx$  (2 mks)

**QUESTION FOUR (20 MARKS)**

- (a) Differentiate with to x
- (i)  $f(x) = \sin(4x+3)$  (3 mks)
- (ii)  $f(x) = \frac{x^2}{\ln x^2}$  (3 mks)
- (b) A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall. (6 mks)
- (c) From first principle (using limits), find the derivative of  $f(x) = \log_e x$  (5 mks)
- (d)  $f(x) = 4x$  (3 mks)

**QUESTION FIVE (20 MARKS)**

- (a) (i) Find  $\frac{dy}{dx}$  if  $x^3 + y^3 = 6xy$  (5 mks)
- (ii) Find the tangent to the following equation  $x^3 + y^3 = 6xy$  at the point (3,3) (2 mks)
- (iii) At what points on the curve is the tangent line horizontal? (3 mks)
- (b) Given  $f(x) = \frac{1}{x}$ , find  $f^{-1}(x)$  (4 mks)
- (c) Find the area under the curve  $y = x^2 - 3x + 4$  and the  $x$  - axis (4 mks)
- (d) The profit function of a product is given by  $p(x) = (100 - x)x - 28x - 54$ . Find the maximum profit of the product. (4 mks)