KABARAK UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **MATH 113**

COURSE TITLE: **CALCULUS I**

STREAM: **SESSION I**

DAY: **TUESDAY**

TIME: **2.00 – 4.00 P.M.**

DATE: **07/04/2009**

INSTRUCTIONS:

Answer **QUESTION ONE** and **ANY OTHER TWO** questions.

PLEASE TURN OVER

Question One (30mks)

(a) Prove that the limit of the sequence.

$Xn = \frac{2n}{3n-1}$ is $\frac{2}{3}$ as $n \to \infty$. Hence

(4mks)

Find the values of N if (i) $\in = 0.03$

(ii) $\in = 0.002$

(iii) $\in = 0.0001$

(6mks)

(b) Using the first principle method find the gradients of the function at the specified point.

(i) y = 4x + 8 at x = 0 **(3mks)**

(ii) $y = \frac{1}{x^2}$ at x = -2 **(3mks)**

(iii) $Y = \sqrt{4X + 4}$ at x = 1 **(3mks)**

(c) Evaluate the following limits.

(i) $\lim\limits_{x \to 0} \frac{x^2 + x}{x}$ **(2mks)**

(ii) $\lim\limits_{n \to \infty} \frac{n^2 + n}{n+2}$ **(2mks)**

(iii) $\lim\limits_{x \to 0} \frac{Cos - Cos3x}{x^2}$ **(2mks)**

(d) Find the derivatives $\frac{dy}{dx}$ of the following functions.

(i) $y = \sqrt{x^2 + 2x + 4}$ **(2mks)**

(ii) $y = x^2 (2x^2 + x + 3)^{-2}$ **(3mks)**

Question Two (20mks)

(a) An object starts from rest and gains an acceleration by a(t) = 6t. What is velocity and distance at t = 7 seconds? **(6mks)**

(b) Find y^1 given $y + 2xy - 1 + y^2 = 0$ **(4mks)**

(c) Find the equations of the tangent and normal lines to the curve $y = 2x^2 + 4x - 3$ at the point where x=1 **(6mks)**

(d) Evaluate $\lim_{x \to 0} \frac{\tan 6x}{8x}$ **(4mks)**

Question Three (20mks)

(a) Show that:

(i) $\frac{d}{dx}$ Sin x = Cos x **(4mks)**

(ii) $\frac{d}{dx}$ Cos x = -Sin x **(4mks)**

(b) Differentiate the following functions w.r.t x

(i) $y = \frac{e^{-ax} + e^{ax}}{e^{ax}}$ **(4mks)**

(ii) $y = \text{Cos}^2 (4x^2) + \text{Sin}^3 2x$ **(3mks)**

(c) Evaluate the following Limit

$$\lim_{x \to -\infty} \left(1 + \frac{3}{x}\right)^{x+4}$$ **(5mks)**

Question Four (20mks)

(a) Using first principle method differentiate $\left(\frac{dy}{dx}\right)$

$y = 4X^2 + 2X + 4$ **(5mks)**

(b) Investigate the local extrema of the function.
$f(x) = 2x^3 - 3x^2 - 12x + 10$ **(5mks)**

(c) The gradient of a curve is 6x – 3. Find the equation of the curve given x – axis is a tangent to the curve. **(4mks)**

(d) Find $\frac{dy}{dx}$ when x = 1 of $y = \frac{u}{u+1}$ and $u = 3x^2 - 1$

(6mks)

Question Five (20mks)

(a) Using $\in -\delta$ definition of a limit verify the following Limit.

$\lim_{x \to 2} (x^3 + x + 1) = 11$ **(7mks)**

(b) Differentiate w.r.t.x

(i) $y = e^{x^2}$ **(2mks)**

(ii) $y = \sin(4x + 5)$ **(3mks)**

(iii) $y = \ln^2(x + 4)$ **(3mks)**

(c) Evaluate the following limits

(i) $\lim_{x \to 25} \frac{\sqrt{x} - 1}{x + 1}$ **(3mks)**

(ii) $\lim_{n \to \infty} \frac{5n^2 + 5n - 2}{3n^3 + 6n^2}$ **(2mks)**