

KABARAK



UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/2010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 113

COURSE TITLE: CALCULUS I

STREAM: SESSION I

DAY: FRIDAY

TIME: 2.00 – 4.00 P.M.

DATE: 27/11/2009

INSTRUCTIONS:

Answer question **ONE** and any other **TWO** questions

PLEASE TURN OVER

QUESTION ONE (30MKS)

(a) Prove that the limit of the sequence.

$$x_n = \frac{2n}{3n-1} \text{ is } \frac{2}{3} \text{ as } n \rightarrow \infty. \text{ Hence}$$

(4mks)

Find the values of N if (i) $\epsilon = 0.01$
(ii) $\epsilon = 0.001$
(iii) $\epsilon = 0.0001$

(6mks)

(b) Using the first principle method find the gradients of the function at the specified point.

(i) $y = 4x + 2$ at $x = 0$ (3mks)

(ii) $y = \frac{1}{x^2}$ at $x = -2$ (3mks)

(iii) $y = \frac{1}{\sqrt{2x+4}}$ at $x = 1$ (3mks)

(c) Evaluate the following limits.

(i) $\lim_{x \rightarrow 0} \frac{x^2 + x}{x}$ (2mks)

(ii) $\lim_{n \rightarrow \infty} \frac{n^2 + n}{n + 2}$ (2mks)

(iii) $\lim_{x \rightarrow 0} \frac{\cos x - \cos 3x}{x^2}$ (2mks)

(d) Find the derivatives $\frac{dy}{dx}$ of the following functions.

(i) $y = \sqrt{x^2 + 2x + 4}$ (2mks)

(ii) $y = x^2 (2x^2 + x + 3)^{-2}$ (3mks)

QUESTION TWO (20MKS)

- (a) An object starts from rest and gains an acceleration by $a(t) = 6t$. What is velocity and distance at $t = 7$ seconds? **(6mks)**
- (b) Find y^1 given $y + 2xy - 1 + y^2 = 0$ **(4mks)**
- (c) Find the equations of the tangent and normal lines to the curve $y = 2x^2 + 4x - 3$ at the point where $x=1$ **(6mks)**
- (d) Evaluate $\lim_{x \rightarrow 0} \frac{\tan 6x}{8x}$ **(4mks)**

QUESTION THREE (20MKS)

- (a) Using $\epsilon - \delta$ definition of a limit verify the following Limit.

$$\lim_{x \rightarrow 2} (x^3 + x + 1) = 11 \quad \textbf{(7mks)}$$

- (b) Differentiate w.r.t.x
- (i) $y = e^{x^2}$ **(2mks)**
- (ii) $y = \sin(4x + 6)$ **(3mks)**
- (iii) $y = \ln^2(x + 3)$ **(3mks)**

- (c) Evaluate the following limits

(i) $\lim_{x \rightarrow 25} \frac{\sqrt{x} - 1}{x + 1}$ **(3mks)**

(ii) $\lim_{n \rightarrow \infty} \frac{4n^2 + 5n - 2}{2n^3 + 3n^2}$ **(2mks)**

QUESTION FOUR (20MKS)

- (a) Compute $\int \left(\frac{x^2}{5x^3 + 1} \right) dx$ **(5mks)**
- (b) Investigate the local extrema of the function.
 $f(x) = 2x^3 - 3x^2 - 12x + 5$ **(5mks)**
- (c) The gradient of a curve is $6x - 3$. Find the equation of the curve given $x - axis$ is a tangent to the curve. **(4mks)**

- (d) Using first principle method differentiate $\left(\frac{dy}{dx}\right)$
 $y = \text{Log}_a X$ **(6mks)**

QUESTION FIVE (20MKS)

- (a) Show that:
- (i) $\frac{d}{dx} \sin x = \cos x$ **(4mks)**
- (ii) $\frac{d}{dx} \cos x = -\sin x$ **(4mks)**
- (b) Differentiate the following functions w.r.t x
- (i) $y = \frac{e^{-ax} + e^{ax}}{e^{ax}}$ **(4mks)**
- (ii) $y = \cos^2(4x^2) + \sin^3 2x$ **(3mks)**
- (c) Evaluate the following Limit

$$\lim_{x \rightarrow -\infty} \left(1 + \frac{3}{x}\right)^{x+4}$$

(5mks)