KABARAK UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN COMPUTER SCIENCE

COURSE CODE: **MATH 113**

COURSE TITLE: **CALCULUS I**

STREAM: **Y1S2**

DAY: **THURSDAY**

TIME: **2.00 – 4.00 P.M.**

DATE: **26/03/2009**

INSTRUCTIONS:

Answer **QUESTION ONE** and **ANY OTHER TWO** questions.

PLEASE TURN OVER

Question One (30mks)

(a) Evaluate the following units

(i) $\lim_{x\to\infty} \frac{(x-6)^{20}(3x+2)^{9}}{(4x^{2}-3)^{15}}$ **(3mks)**

(ii) $\lim_{n\to\infty} \frac{n^{2}+4n+3}{3n^{2}+7}$ **(2mks)**

(iii) $\lim_{x\to\infty} \frac{x^{2}-4}{(x-2)(x-1)}$ **(2mks)**

(iv) $\lim_{x\to 0} \frac{\cos x-\cos 3x}{x^{2}}$ **(3mks)**

(b) Find $\frac{dy}{dx}$ of the following functions from the first principles

(i) $y=3x^{2}+2x+1$ at $x=1$ **(3mks)**

(ii) $y=\frac{1}{x^{2}}$ at $x=2$ **(3mks)**

(c) Let $\lim_{x\to\infty} x_n = A$ and $\lim_{x\to\infty} y_n = B$ then show that

(i) $\lim_{x\to\infty} x_n + y_n = A+B$ **(3mks)**

(ii) $\lim_{n\to\infty} x_n\, y_n = A\cdot B$ **(3mks)**

(d) Using ε - δ definition of limits show that;

$$\lim_{x\to 1} 2x^{2}+4x+7=13$$

Hence evaluate the value of δ when

(i) $\varepsilon = 0.1$ at $x=1$

(ii) $\varepsilon = 0.03$ at $x=2$ **(8mks)**

Question Two (20mks)

(a) Find $\left(\frac{dy}{dx}\right)$ of the following functions

(i) $y = (2x^2 + 4)(x^4 + 2x + 3)$ **(3mks)**

(ii) $y = (6x^2 + 2x + 4)^{10}$ **(2mks)**

(iii) $2x^2 y + 4xy + x + 8 = 0$ **(4mks)**

(b) Evaluate the following limits

(i) $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x+4}$ **(3mks)**

(ii) $\lim_{x \to 0} \frac{\sin x + \cos x}{x^2}$ **(4mks)**

(c) Verify that $\lim_{x \to 1} 2x^2 + 4 + 2 = 8$ **(4mks)**

Question Three (20mks)

(a) Find $\left(\frac{dy}{dx}\right)$ of the following functions

(i) $y = \sin 3x$ **(3mks)**

(ii) $y = \cos 4x + \sin 3x$ **(3mks)**

(iii) $y = e^x \sin^2 x + 4\cos x$ **(3mks)**

(iv) $y = \frac{e^{ax} + e^{-ax}}{e^x}$ **(4mks)**

(b) From first principles show that

$$\frac{d}{dx}(\cos x) = -\sin x$$ **(7mks)**

Question Four (20mks)

(a) Prove that the limit of the sequence $X_n = \frac{3n}{2n-1}$ is $\frac{3}{2}$ as $n \to \infty$

stating the values of N such that n>N we shall have the inequality $\left|x_n - \frac{3}{2}\right| < \varepsilon$ valid for any arbitrary $\varepsilon > 0$ then find the values of N if

(i) $\varepsilon = 0.03$

(ii) $\varepsilon = 0.003$ **(10mks)**

(b) (i) Find the velocity and acceleration at the time t = 2 for a particle moving in a straight line if its motion obeys the law

$S = t^3 + 5t^2 + 4$ **(4mks)**

(ii) When is the particle stationary? **(3mks)**

(c) Find $\lim\limits_{x \to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$ **(3mks)**

Question Five (20mks)

(a) Write the equations of tangent and normal to the curve $x^3 - y^2 - 2xy = 0$ at a point (1, 2) **(5mks)**

(b) Determine whether $y = A\cos ax + B\sin ax$ satisfies $y^{11} + a^2 y = 0$ **(5mks)**

(c) Investigate the local extrema of the curve $y = 2x^2 + 6x + 2$ **(5mks)**

(d) Find the area enclosed by the curve $y = x^2 - 16$ **(5mks)**