KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2010/2011 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 113

COURSE TITLE: CALCULUS I

STREAM: SESSION I

DAY: FRIDAY

TIME: 2.00 – 4.00 PM

DATE: 26/11/2010

INSTRUCTIONS:

- Answer question **ONE** and any other **TWO** questions

PLEASE TURNOVER

QUESTION ONE (30MKS)

(a) Prove that the limit of the sequence.

$X_n = \frac{2n}{3n-1}$ is $\frac{2}{3}$ as $n \to \infty$. Hence

(4mks)

Find the values of N if (i) $\in = 0.01$

(ii) $\in = 0.001$

(iii) $\in = 0.0001$

(6mks)

b)Show that $\lim_{x \to a} x^2 = a^2$ **(5 mks)**

(c) Evaluate the following limits.

(i) $\lim_{x \to 0} \frac{x^2 + x}{x}$ **(3mks)**

(ii) $\lim_{n \to \infty} \frac{n^2 + n}{n + 2}$ **(3mks)**

(iii) $\lim_{x \to 0} \frac{Cos - Cos 3x}{x^2}$ **(3mks)**

(d) Find the derivatives $\frac{dy}{dx}$ of the following functions.

(i) $y = \sqrt{x^2 + 2x + 4}$ **(2mks)**

(ii) $y = x^2 (2x^2 + x + 3)^{-2}$ **(3mks)**

QUESTION TWO (20MKS)

(a) An object starts from rest and gains an acceleration by a(t) = 6t. What is velocity and distance at t = 7 seconds? **(6mks)**

(b) Find y^1 given $y + 2xy - 1 + y^2 = 0$ **(4mks)**

(c) Find the equations of the tangent and normal lines to the curve $y = 2x^2 + 4x - 3$ at the point where x=1 **(6mks)**

(d) Evaluate $\lim_{x \to 0} \frac{\tan 6x}{8x}$ **(4mks)**

QUESTION THREE (20MKS)

(a) Using $\in -\delta$ definition of a limit verify the following Limit.

$$\lim_{x\to 2} (x^3 + x + 1) = 11$$ **(7mks)**

(b) Differentiate w.r.t.x

(i) $y = e^{x^2}$ **(2mks)**

(ii) $y = \text{Sin}(4x + 6)$ **(3mks)**

(iii) $y = \text{Ln}^2 (x + 3)$ **(3mks)**

(c) Evaluate the following limits

(i) $\lim_{x\to 25} \frac{\sqrt{x}-1}{x+1}$ **(3mks)**

(ii) $\lim_{n\to\infty} \frac{4n^2+5n-2}{2n^3+3n^2}$ **(2mks)**

QUESTION FOUR (20MKS)

(a) Compute $\int\left(\frac{x^2}{5x^3+1}\right)dx$ **(5mks)**

(b) Investigate the local extrema of the function.
$f(x) = 2x^3 - 3x^2 - 12x + 5$ **(5mks)**

(c) The gradient of a curve is $6x - 3$. Find the equation of the curve given x – axis is a tangent to the curve. **(4mks)**

(d) Using first principle method differentiate $\left(\frac{dy}{dx}\right)$

$y = \text{Log}_a X$ **(6mks)**

QUESTION FIVE (20MKS)

(a) Show that:

(i) $\frac{d}{dx}$ Sin x = Cos x **(4mks)**

(ii) $\frac{d}{dx}$ Cos x = -Sin x **(4mks)**

(b) Differentiate the following functions w.r.t x

(i) $y = \frac{e^{-ax} + e^{ax}}{e^{ax}}$ **(4mks)**

(ii) $y = \text{Cos}^2 (4x^2) + \text{Sin}^3 2x$ **(3mks)**

(c) Evaluate the following Limit

$$\underset{x \to -\infty}{Lim} \left(1 + \frac{3}{x} \right)^{x+4}$$ **(5mks)**