KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF COMPUTER SCIENCE

COURSE CODE: MATH 113

COURSE TITLE: CALCULUS I

STREAM: Y1S1

DAY: **THURSDAY**

TIME: **8.30 – 10.30 A.M**

DATE: **11/12/2008**

INSTRUCTIONS TO CANDIDATES:

1. Answer Question **ONE** and any other **TWO** Questions

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) Prove that the limit of the function $f(\mathrm{x}) = 3x+5$ as $x \to 3$ is 14, stating the value of ∂ such that for all $\varepsilon > 0$ we shall have the inequality $|f(x)-14| < \varepsilon$ valid. Find the values of ∂ if

(i) $\varepsilon = 0$

(ii) $\varepsilon = 0.5$

(iii) $\varepsilon = 0.01$ **(7 mks)**

(b) Find the derivative $\left(\frac{dy}{dx}\right)$ of the following functions from first principles technique.

(i) $y = 2x^2 + 4x$ **(3 mks)**

(ii) $y = \sqrt{x+2}$ **(3 mks)**

(c) Evaluate the following limits

(i) $\lim\limits_{x \to \infty} \dfrac{x^2 - x - 1}{(2x^2 + x - 1)}$ **(2**

(ii) $\lim\limits_{\chi \to 3} \dfrac{x^2 + 5\chi + 6}{\chi^2 + 8x + 15}$ **(2 mks)**

(d) Find the derivative $\frac{dy}{dx}$ of the following functions.

(i) $y = (4x^2 + 10)^{20}$ **(2 mks)**

(ii) $y = (2x^2 + 4x + 1)(x+3)^{10}$ **(3 mks)**

(e) Compute the indefinite integral

$$\int \frac{x^2}{5x^2+1} dx$$ **(4 mks)**

(f) Find the derivative $\frac{dy}{dx}$ of the function expressed implicitly;

$$\chi^3 + y^2 + 2xy - 6 = 0$$ **(4 mks)**

QUESTION TWO (20 MARKS)

(a) Find the derivatives of the following functions;

(i) $y = 2^{\sin x}$ **(3 mks)**

(ii) $y = \frac{x^2}{\ln x}$ **(3 mks)**

(b) Investigate the local extrema of the function

$f(x) = x^4 - 8x^2 + 10$ **(4 mks)**

(c) Integrate

(i) $\int (3x+2)^{2S}$ **(3 mks)**

(ii) $\int_{\sqrt{5}}^{2\sqrt{2}} \frac{xdx}{\sqrt{3x^2+1}}$ **(4 mks)**

(d) Given $y = \frac{1}{x}$ and $x > 0$, use a sketch to show the two limiting values of the sequence.

(3 mks)

QUESTION THREE (20 MKS)

(a) Prove that the limit of the sequence Xn $= \frac{3n}{2n-1}$ is $\frac{3}{2}$ as $n \to \infty$

where n = 1, 2, 3, ------)

stating the values of N such that $n > N$ we shall have the inequality $\left|X_n - 3/2\right| < \varepsilon$ valid for any arbitrary $\varepsilon > 0$ Find the values of N if

(i) $\varepsilon = 0.03$

(ii) 0.003

(iii) $\varepsilon = 0.0003$

Explaining where those terms can be found. **(10 mks)**

(b) Evaluate the limits (explaining every fact you use)

(i) $\lim_{x \to 0} \frac{\tan x}{x}$ **(3 mks)**

(ii) $\lim_{x \to \infty} \left(\frac{3x+1}{3x-2}\right)^{2x}$ **(4 mks)**

(iii) $\lim_{x \to 0} \frac{e^{x} - e^{-x} - 2x}{x - \sin x}$ **(3 mks)**

<u>QUESTION FOUR (20 MKS)</u>

(a) (i) Find the velocity and acceleration at the time t = 2 for a particle moving in a straight line if its motion obeys the law

$S = t^3 + 5t^2 + 4$ **(4 mks)**

(ii) When is the particle stationary? **(3 mks)**

(b) Find $y^1 = \frac{dy}{dx}$ given $y = a^{3x^2}$ **(3 mks)**

(c) Find $\frac{dy}{dx}$ if $y = e^{-2x} \sin 3x$ **(5 mks)**

(d) Write the equations of tangent and normal to the curve $x^3 - y^2 - 2xy = 0$ at a point (1, 2) **(5 mks)**

QUESTION FIVE (20 MARKS)

(a) (i) Determine whether or not the function

$Y = A\cos ax + BSin\, ax$ Satisfies the equation

$Y^{11} + a^2 y = 0$ **(4 mks)**

(ii) Show that if the motion of a particle obeys the law; S(t) = $ae^{t} + be^{-t}$ the acceleration is equal to its displacement (4mks)

(b) Find the derivative $\frac{dy}{dx}$ of the function

$$y = \frac{e^{ax} - e^{-ax}}{e^{ax} + e^{-ax}}$$ **(7 mks)**

(c) Prove that $\frac{d}{dx}(Sinx) = \cos x$ **(5 mks)**