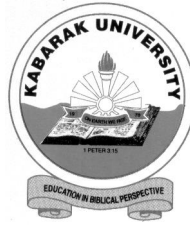


KABARAK



UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

**FOR THE DEGREE OF BACHELOR OF EDUCATION
SCIENCE**

COURSE CODE: MATH 113
COURSE TITLE: CALCULUS I
STREAM: SESSION I & III
DAY: THURSDAY
TIME: 9.00 – 11.00 A.M
DATE: 25/11/2008

INSTRUCTIONS TO CANDIDATES:

1. Answer Question **ONE** and any other **TWO** Questions

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) Differentiate the following function from first principles, $f(x) = \frac{1}{x+1}$ (4 mks)

(b) Use differential to find the approximate value of $(33)^{\frac{2}{5}}$. (5 mks)

(c) Evaluate the following limits

(i) $\lim_{x \rightarrow \infty} \frac{x^3}{(1+x^3)}$ (2 mks)

(ii) $\lim_{\chi \rightarrow 0} \frac{\sin 2\chi}{\chi^2 + x}$ (2 mks)

(d) Differentiate the following functions with respect to χ

(i) $y = 9^{-x}$ (3 mks)

(ii) $y = \tan(\cos x^2)$ (3 mks)

iii) $y = \sin^{-1} \frac{\chi-3}{\chi+3}$ (4 mks)

(e) i) State two conditions for a function $f(\chi)$ to be continuous at a point $\chi = a$ (2 mks)

(i) Let $f(x) = \begin{cases} \frac{x^3-4}{x-2} & \chi \neq 2 \\ 3 & \chi = 2 \end{cases}$

Show that $f(x)$ is discontinuous at $x=2$. What value should $f(x)$ have in order to be continuous at $x = 2$? (5 mks)

QUESTION TWO (20 MARKS)

(a) Find the value of the stationary point of

$$f(x) = \frac{x}{x^2+2} \quad (13 \text{ mks})$$

(b) Find the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point (1,1) on the curve $3xy + y^2 - x - y = 0$ **(7 marks)****QUESTION THREE (20 MKS)**

(a) Given the parametric equations

$$x = \frac{at}{1-t^2} \quad y = \frac{at^2}{1-t} \quad \text{find } \frac{dy}{dx} \text{ in terms of } t.$$

(b) Given that $y = \ln \sqrt{\frac{(1-x^2)}{(1+x^2)}}$ show that $\frac{dy}{dx} = \frac{-2x}{(1-x^4)}$ (c) Given that $y = e^{-2t} \cos 3t$ show that $\frac{d^2y}{dt^2} + 4 \frac{dy}{dt} + 13y = 0$ **QUESTION FOUR (20 MKS)**(a) Find the equations of the tangent and normal to the curve $y = e^{-3x} + 5x - 5$ at the point (0, 4). **(7 mks)**(b) A particle moving in a straight line is a distance s metres from a fixed point after t seconds, where $S = t^3 - 4t^2 + 5t$.Find an expression for the speed V and the acceleration, a , after t seconds. For what values of t in the particle stationary and what is the acceleration at these times.**(8 mks)**

(c) i) State the Rolle's Theorem

ii) Let $f(x) = x^{2/3}$ on $(-8, 27)$. Show that the Mean Value Theorem fails.**(5 mks)**

QUESTION FIVE (20 MARKS)

(a) The average value of a function f over the interval (a,b) is given by

$$\frac{\int_a^b f(x)dx}{b-a}$$

Determine the average value of the function $f(x) = x^2 + 3x + 1$ over the interval $(-1,1)$.
(5 mks)

(b) Find the indefinite integrals

i) $\int 3x \sqrt{3x^2 + 7} dx$

ii) $\int \frac{x}{x^2 + a^2} dx$ **(6 mks)**

(c) Evaluate the definite integrals

(i) $\int_0^{\pi} 3 \sin x dx$

(ii) $\int_0^4 (2x+1) \sqrt{x^2 + x} dx$

(iii) $\int_0^{\pi/4} \sin^3 2x \cos 2x dx$ **(9 marks)**