KABARAK  **UNIVERSITY**

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 113

COURSE TITLE: CALCULUS I

STREAM: SESSION I & III

DAY: **THURSDAY**

TIME: **9.00 – 11.00 A.M**

DATE: **25/11/2008**

INSTRUCTIONS TO CANDIDATES:

1. Answer Question **ONE** and any other **TWO** Questions

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) Differentiate the following function from first principles, $f(x) = \frac{1}{x+1}$ **(4 mks)**

(b) Use differential to find the approximate value of $(33)^{2/5}$. **(5 mks)**

(c) Evaluate the following limits

(i) $\lim_{x \to \infty} \frac{x^3}{(1+x^3)}$ **(2 mks)**

(ii) $\lim_{\chi \to 0} \frac{Sin2\chi}{\chi^2 + x}$ **(2 mks)**

(d) Differentiate the following functions with respect to χ

(i) $y = 9^x$ **(3 mks)**

(ii) $y = \tan(\cos x^2)$ **(3 mks)**

iii) $y = \sin^{-1} \frac{\chi - 3}{\chi + 3}$ **(4 mks)**

(e) i) State two conditions for a function $f(\chi)$ to be continuous at a point $\chi = a$ **(2 mks)**

(i) Let $f(x) = \begin{cases} \left(\frac{x^3 - 4}{x-2}\right) & \chi \neq 2 \\ 3 & \chi = 2 \end{cases}$

Show that f (x) is discontinuous at x=2. What value should f (x) have in order to be continuous at x = 2? **(5 mks)**

QUESTION TWO (20 MARKS)

(a) Find the value of the stationary point of

$$f(x) = \frac{\chi}{\chi^2+2}$$ **(13 mks)**

(b) Find the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point (1,1) on the curve $3xy + y^2 - x - y = 0$

(7 marks)

QUESTION THREE (20 MKS)

(a) Given the parametric equations

$$x = \frac{at}{1-t^2} \quad y = \frac{at^2}{1-t}$$ find $\frac{dy}{dx}$ in terms of t.

(b) Given that $y = \text{in}\sqrt{\frac{(1-x^2)}{(1+x^2)}}$ show that $\frac{dy}{dx} = \frac{-2x}{(1-x^4)}$

(c) Given that $y = e^{-2t}\cos 3t$ show that $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 13y = 0$

QUESTION FOUR (20 MKS)

(a) Find the equations of the tangent and normal to the curve $y = e^{-3x} + 5x - 5$ at the point (0, 4). **(7 mks)**

(b) A particle moving in a straight line is a distance s metres from a fixed point after t seconds, where $S = t^3 - 4t^2 + 5t$.
Find an expression for the speed V and the acceleration, a, after t seconds. For what values of t in the particle stationary and what is the acceleration at these times.
(8 mks)

(c) i) State the Rolle's Theorem

ii) Let $f(x) = \chi^{2/3}$ on (-8, 27). Show that the Mean Value Theorem fails.
(5 mks)

QUESTION FIVE (20 MARKS)

(a) The average value of a function f over the interval (a,b) is given by

$$\frac{\int_a^b f(x)dx}{b-a}$$

Determine the average value of the function f (x) = x^2 + 3x + 1 over the interval (-1,1). **(5 mks)**

(b) Find the indefinite integrals

i) $\int 3x\ \sqrt{3x^2} + 7\ dx$

ii) $\int \frac{x}{x^2+a^2}\,dx$ **(6 mks)**

(c) Evaluate the definite integrals

(i) $\int_0^{\pi} 3\sin x\,dx$

(ii) $\int_0^4 (2x+1)\sqrt{x^2}+x\ dx$

(iii) $\int_0^{\pi/4} \sin^3 2x \cos 2x\,dx$ **(9 marks)**