# KABARAK UNIVERSITY

## UNIVERSITY EXAMINATIONS

### 2009/2010 ACADEMIC YEAR

### FOR THE DEGREE OF BACHELOR OF COMPUTER SCIENCE AND BACHELOR OF SCIENCE IN ECONOMICS AN D MATHEMATICS

**COURSE CODE:** **MATH 113**

**COURSE TITLE:** **DIFFERENTIAL CALCULUS I**

**STREAM:** **Y1S1**

**DAY:** **THURSDAY**

**TIME:** **2.00 – 4.00 P.M.**

**DATE:** **10/12/2009**

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**INSTRUCTIONS:**

1. Attempt Question **ONE** and any other **TWO** Questions
2. Show **ALL** your workings.

**PLEASE TURN OVER**

## QUESTION ONE (30 MARKS)

(a) Given two points c,d$\in R$ and we set c<d, state the four possible intervals which can be deduced from the setting (4 mks)

(b) Using the inequality signs, where possible, represent the following intervals

(i) $(-\infty, +\infty)$

(ii) $(-\infty, a)$

(iii) $(a +\infty)$

(iv) $[a, +\infty)$ (5 mks)

(c) Evaluate the following limits

(i) $\lim_{x\to 0} \frac{x^2+x}{x}$ (3 mks)

(ii) $\lim_{x\to 1} \frac{x^2+5x+6}{x^2-4}$ (3 mks)

(iii) $\lim_{n\to\infty} \frac{n^2+2n+1}{2n^2+4}$ (3 mks)

(d) Find the derivative $\left(\frac{dy}{dx}\right)$ of the following functions from first principles

(i) $y = 3x+8$ (3 mks)

(ii) $y = \frac{1}{x^2}$ (4 mks)

(e) Using the definition of limit and epsilon show that $a_n = \frac{n}{2n+1} \to \frac{1}{2}$ and hence find the value of $N(\in)$ if $\in = 0.03$. (3 mks)

(e) Differentiate $\left(\frac{dy}{dx}\right)$ the function

$f(x) = \sqrt{2x^2 + 4x + 4}$ (2 mks)

## QUESTION TWO (20 MARKS)

(a) Differentiate the following functions w,r,t,x and find gradients at specific points

(i) $y = (2x^2 + 2x + 3)(x^2 + 1)$ at (0,2) (3 mks)

(ii) $y = (\frac{(2x+3)^3}{(2x-3)^2}$ at (1,4) (3 mks)

(iii) $2xy + y^2 + 3 = 0$ at (-1,7) (4 mks)

(iv) $y = e^{x} \sin 3x$ at (0,11) (3 mks)

(b) Evaluate the following limits

(i) $\lim_{x\to 0} \frac{\tan 6x}{8x}$ (2 mks)

(ii) $\lim_{x\to 9} \frac{6-\sqrt{x}}{6-x}$ (2 mks)

(iii) $\lim_{x\to 0} \frac{\text{Cos } x - \text{Cos } 3x}{x^2}$ (3 mks)

## QUESTION THREE (20 MKS)

(a) Find the velocity and acceleration at the time t=3 seconds for a particle moving in a straight line if its motion obeys the law $S = t^3 + 5t^2 + 4$ (4 mks)

(b) Find the derivatives $\left(\frac{dy}{dx}\right)$ of the functions

(i) $f(x) = \cos^2 2x + \sin^3 3x$ (3 mks)

(ii) $y = \ln^2 \sin x$ (3 mks)

(c) Verify the following limits

(i) $\lim_{x\to a} x^2 = a^2$ (5 mks)

(ii) $\lim_{x\to -1} 4x + 2 = -2$ (5 mks)

## QUESTION FOUR (20 MKS)

(a) From first principles show that

(i) $\frac{d}{dx} \cos x = -\sin x$ (5 mks)

(ii) $\frac{d}{dx} \sin x = \cos x$ (5 mks)

(b) Given y = $\frac{e^{ax} + e^{-ax}}{e^{ax} - e^{-ax}}$ find $y^{11}$ (6 mks)

(c) Evaluate $\lim_{x \to \infty} \left(1 + \frac{4}{x}\right)^{x+4}$ (4 mks)

**QUESTION FIVE (20 MARKS)**

(a) Find the equation of tangent and normal to the curve $x^2 - y^2 = 7$ at a point (4,3) (6 mks)

(b) Investigate the local extrema of the function y = $2x^3 - 3x^2 - 12x + 5$ (4 mks)

(c) Verify the following limits

(i) $\lim_{x \to a} x^2 = a^2$ (4 mks)

(ii) $\lim_{x \to 1} x^2 + 4x + 3 = 8$

(d) Find the equation of the curve given the gradient is 3x – 2 at a point (4,2) (2 mks)