KABARAK   **UNIVERSITY**

UNIVERSITY EXAMINATIONS

2009/2010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 113

COURSE TITLE: CALCULUS I

STREAM: SESSION I

DAY: FRIDAY

TIME: 9.00 – 11.00 A.M.

DATE: 09/04/2010

INSTRUCTIONS:

Attempt question **ONE** and any other **TWO** Questions

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a)   Given that $xy = \sin^{-1} \ln|2x^2 + x|$

Find $\dfrac{dy}{dx}$ **(4 marks)**

(b)   Using the first principle method, find the gradients of the function at specified point.

(i) $y = 4x + 2$ at $x = 0$ **(2 marks)**

(ii) $y = \dfrac{1}{\sqrt{2x + 4}}$ at $x = 1$ **(3 marks)**

(iii) $y = e^{-2x}$ at $x = 0$ **(2 marks)**

(c)   Evaluate the following limits

(i) $\text{Lim}_{x \to 0} \ \dfrac{x^2 + x}{x}$ **(1 mark)**

(ii) $\text{Lim}_{x \to 0} \ \dfrac{\cos x - \cos 3x}{x^2}$ **(3 marks)**

(d)   Differentiate the following with respect to x

(i) $y = (x - 1)\sqrt{x^2 - 2x + 2}$ **(3 marks)**

(ii) $y = \left(\dfrac{x^3 - 1}{2x^3 + 1}\right)^4$ **(2 marks)**

(iii) $y = \dfrac{e^{2x} \cos x}{\ln x}$ **(3 marks)**

(e)   If $y = x^2 - 4x$ and $x = \sqrt{2t^2 + 1}$

Find $\dfrac{dy}{dx}$ when $t = \sqrt{2}$ **(2 marks)**

(f)   At a given instant the radii of two concentric circles are 8 cm and 12 cm. The radius of the outer circle increases at the rate of Km/s and that of the inner circle at the rate of 2 cm/s. Find the rate of change of the area enclosed between the two circles. **(3 marks)**

QUESTION TWO (20 MARKS)

(a) The path of a particle moving in a straight line is given by $s = \frac{t^3}{3} - t^2 - 3t + 4$.

Find the distance(s) and acceleration when velocity is zero. **(3 marks)**

(b) Find the value of $\frac{d^2y}{dx^2}$ at the point $(-1, 1)$ of the curve $x^2y + 3y - 4 = 0$ **(7 marks)**

(c) Find the equations of the tangent and normal lines to the curve $y = 2x^2 + 4x - 3$ at the point where $x = 1$ **(5 marks)**

(d) Evaluate $\lim_{x \to 0} \frac{\tan 6x}{8x}$ **(3 marks)**

(e) Given that $y = \sqrt{1 + \sqrt{x}}$, Find $\frac{dy}{dx}$ **(2 marks)**

QUESTION THREE (20 MARKS)

(a) Differentiate the following with respect to x

(i) $y = x\sqrt{1 + \sqrt{4x + 2}}$ **(3 marks)**

(ii) $y = \frac{x \ln 2x}{\sin x}$ **(3 marks)**

(iii) $y = 3x^2 \cos x$ **(3 marks)**

(b) Given that $y = \frac{u^2 + 1}{u^2 - 1}$ and $u = \sqrt{2x^3 - 5}$ Find $\frac{dy}{dx}$ **(5 marks)**

(c) Evaluate the following limits

(i) $\lim_{x \to 0} \frac{xe^{2x} + 1}{x^2}$ **(3 marks)**

(ii) $\lim_{x \to \infty} \frac{4n^2 + 5n - 2}{2n^3 + 3n^2}$ **(3 marks)**

QUESTION FOUR (20 MARKS)

(a)   Investigate the turning points of the function
$$f(x) = 2x^3 - 3x^2 + 12x + 5$$
(5 marks)

(b)   The gradient of a curve is $6x - 3$. Find the equation of the curve given x - axis is a tangent to the curve.   **(4 marks)**

(c)   Find the slope of the curve $x = y^2 - 4y$ at the points where it crosses the y − axis.

(4 marks)

(d)   A point moves in the plane according to the law $x = t^2 + 2t$ and $y = 2t^3 - 6t$.

Find $\frac{dy}{dx}$ where $t = 0$ **(5 marks)**

(e)   Differentiate $y = xe^{x^2}$ with respect to x **(2 marks)**

QUESTION FIVE (20 MARKS)

(a)   Show that;

(i) $\frac{d}{dx} \sin x = \cos x$ **(4 marks)**

(ii) $\frac{d}{dx} \cos x = -\sin x$ **(4 marks)**

(b)   Differentiate the following functions with respect to x

(i) $y = \frac{e^{-ax} + e^{ax}}{e^{ax}}$ **(4 marks)**

(ii) $y = \cos^2(4x^2) + \sin^2 2x$ **(3 marks)**

(c)   Find $\frac{dy}{dx}$ given that $y = t\ln(t^2 + 1)$ and $x = t^2 e^{2t}$ **(5 marks)**