KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/2010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF COMPUTER SCIENCE AND BACHELOR OF ECONOMIC & MATHEMATICS

COURSE CODE: **MATH 113**

COURSE TITLE: **CALCULUS I**

STREAM: **Y1S2 & Y1S1**

DAY: **WEDNESDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **24/03/2010**

INSTRUCTIONS:

Attempt question **ONE** and any other **TWO** Questions

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) Using first principle technique find $\frac{dy}{dx}$ of the following functions;

(i) $f(x) = e^{-2x}$ at $x = 0$ **(4 marks)**

(ii) $f(x) = 3x^3 + 2x^2 + 4x + 7$ at $x = 1$ **(5 marks)**

(b) Evaluate the following limits explaining every step:

(i) $\text{Lim}_{x \to \sqrt{2}} \frac{x^4 + 4}{x^2 - 2}$ **(3 marks)**

(ii) $\text{Lim}_{x \to \infty} \frac{x^2 - 3x + 3}{2x^2 + 4}$ **(3 marks)**

(iii) $\text{Lim}_{x \to 0} \frac{x^2 \sin 2x}{3x}$ **(4 marks)**

(c) Find y^1 given $y + 2xy - 1 + y^2 = 0$ **(5 marks)**

(d) Find the equations of the tangent and the normal lines to the curve $x^2 - y^2 = 7$ at a point $(4, 3)$ **(6 marks)**

QUESTION TWO (20 MARKS)

(a) The profit P is the revenue minus the cost C. If the total revenue from selling n bicycles is £50 n – $0.1n^2$ and the cost of manufacturing n bicycles is £ $30n + 500 - 0.025n^2$.
What output will maximize profit? What is the maximum profit? **(7 marks)**

(b) Find y^1 given $y = \frac{3x^2 - 6x + 7}{4x}$ **(4 marks)**

(c) Give the precise, i.e. ε - δ, definition of the continuity of a function. **(2 marks)**

(d) Use the precise definition of a limit to show that; $\lim_{x \to -1} x^3 + x + 1 = 11$ **(7 marks)**

QUESTION THREE (20 MARKS)

(a) Find the equation of the tangent line to the graph of $x^4 + xy^3 + y^4 = 3$ at the point $(1, 1)$. **(7 marks)**

(b) Evaluate $\text{Lim}_{x \to -1} \frac{x+1}{x^2+4x+3}$ **(3 marks)**

(c) Find y^1 if $y = \sin(3x^2 + 2)^3$ **(4 marks)**

(d) Find y^{11}, given $y = e^{-2x} \sin 3x$ **(6 marks)**

QUESTION FOUR (20 MARKS)

(a) State the coordinates and nature of the turning points on the curve
$y = x^3 - 5x^2 + 3x + 2$ **(8 marks)**

(b) A particle is travelling along the x-axis and its position at time t is given by $x(t) = t^2 + 3t - 1$. Determine its velocity $V(t)$ and acceleration $a(t)$ when $t = 5$ **(4 marks)**

(c) Compute $f^1(x)$ at $x = 1$ from first principles if $f(x) = 3x^2 + 1$ **(4 marks)**

(d) Given $f(x) = 4x^2 + 3x + 1$ and $g(x) = 2x - 1$, find:
(i) $(f + g)(-2)$ **(2 marks)**
(ii) $(f \cdot g)(-2)$ **(2 marks)**

QUESTION FIVE (20 MARKS)

(a) Evaluate $\int (5 \sin x - 3x^4) dx$ **(3 marks)**

(b) Show that $\frac{d}{dx}(\cos x) = -\sin x$ **(5 marks)**

(c) Differentiate, with respect to x, $In(\cos 3x)$ **(3 marks)**

(d) Determine and distinguish the stationary points of the curve
$y = x^3 - 6x^2 + 9x + 2$ **(6 marks)**

(e) Find the derivative of $f(x) = \sqrt[3]{x^4 + 3x^2 + 5}$ **(3 marks)**