KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/20010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS AND BACHELOR OF COMPUTER SCIENCE

COURSE CODE: **MATH 113**

COURSE TITLE: **CALCULUS I**

STREAM: **Y1S1**

DAY: **THURSDAY**

TIME: **2.00 – 4.00 P.M.**

DATE: **12/08/2010**

INSTRUCTIONS:

- Attempt question **ONE** and any other **TWO** Questions

PLEASE TURNOVER

QUESTION ONE (30 MARKS)

(a) Using the precise definition of limits show that

$\lim_{n\to\infty}\left\{\frac{(-1)^n}{n}\right\} \to 0$ where $n \in N$. Hence evaluate $N(\in)$ when

(i) $\varepsilon = 0.01$

(ii) $\varepsilon = 0.03$

(iii) $\varepsilon = 0.001$ **(6 marks)**

(b) Explaining every step evaluate the following limits

(i) $\text{Lim}_{n\to\infty} \frac{n^2 - 4n + 7}{2n^2 + 4}$ **(2 marks)**

(ii) $\lim_{x\to 0} \frac{\sqrt[m]{1+ax} - \sqrt[n]{1+bx}}{x}$ **(2 marks)**

(iii) $\lim_{x\to\infty} \frac{(x-3)^{40}\,(5x+6)^{10}}{(3x^2-6)^{25}}$ **(2 marks)**

(iv) $\lim_{x\to 0} \frac{x^x \sin 2x}{3x}$ **(2 marks)**

(c) From the first principles find $\frac{dy}{dx}$ of the following functions;

(i) $f(x) = \frac{1}{x^2}$ **(3 marks)**

(ii) $f(x) = \sqrt{x+2}$ **(3 marks)**

(d) Let $\lim_{n\to\infty} X_n \to A$ and $\lim_{n\to\infty} y_n \to B$ Show that

(i) $\lim_{n\to\infty} x_n + y_n = A + B$ **(5 marks)**

(ii) $\lim_{n\to\infty} x_n\, y_n = A \cdot B$ **(5 marks)**

QUESTION TWO (20 MARKS)

(a) Find y' and y'' given $y = \frac{e^{ax} + e^{-ax}}{e^{ax} - e^{-ax}}$ **(6 marks)**

(b) Find y'' given $y = e^{-2x}\sin 3x$ **(5 marks)**

(c) Find y' and y'' given $x^4 + xy^3 + y^3 = 3$ at the point $(1, 1)$ **(5 marks)**

(d) Show that $\lim_{x\to 2} x^2 + 2x + 2 = 10$ **(4 marks)**

QUESTION THREE (20 MARKS)

(a) A particle moves along a straight line in such away that its distance from a fixed point on the line after t seconds is S meters, where $S = \frac{1}{6}t^4$. Find;

(i) Its velocity after 3 seconds and 4 seconds

(ii) Its acceleration after 2 seconds and 4 seconds **(5 marks)**

(b) A ball was thrown upwards with a velocity of 40 m/s. Find

(i) The acceleration, velocity and distance expressions.

(ii) The maximum height the ball can attain (strictly use calculus techniques) **(6 marks)**

(c) The volume of a cylindrical tank is $16\pi mls$. Find the radius of the tank if the area has to be least. **(5 marks)**

(d) A 100 meter wire mesh was provided to fence a rectangular plot. Find the maximum area it can enclose by mesh without any loss. **(4 marks)**

QUESTION FOUR (20 MARKS)

(a) Use the first principle to find $\frac{dy}{dx}$ of the following functions

(i) $y = \sin x$ **(4 marks)**

(ii) $y = \cos x$ **(4 marks)**

(b) Given $y = 2x^3 - 15x^2 + 24x + 19$ find the stationary points. **(4 marks)**

(c) Differentiate the following functions.

(i) $y = \text{Sin}^3 2x$ **(2 marks)**

(ii) $y = \frac{Tan^2 e^x\{\cos x\}}{x^2}$ **(2 marks)**

(d) Evaluate (i) $\text{Lim}_{x \to \infty} \left\{\frac{(3x+1)}{3x-2}\right\}^{2x}$ **(2 marks)**

(ii) $\text{Lim}_{x \to 0} \frac{1-\cos 3x}{x^2}$ **(2 marks)**

QUESTION FIVE (20 MARKS)

(a) Given $y = \frac{\cos x}{x}$, prove that $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + xy = 0$

(7 marks)

(b) Evaluate $\int \frac{x^2}{5x^2 + 1}\, dx$ **(5 marks)**

(c) Determine whether $y = Ae^{ax} + Be^{-ax}$ is satisfied by $y'' - a^2y = 0$ **(3 marks)**

(d) Find the equation of the curve given the gradient is $4x - 2$ and $x -$ axis is the tangent.

(5 marks)