

QUESTION ONE (30 MARKS)

- a) Use the trapezoidal rule with $n = 4$ to estimate the following integral and compare with the exact value

$$\int_1^2 x^2 dx$$

(5 marks)

- b) Evaluate the following functions

i) $\int \cos(7\theta + 5) d\theta$

ii) $\int \sqrt{\tan x} \sec^2 x dx$

(9 marks)

iii) $\int x e^{-x^2} dx$

- c) Evaluate the following inverse functions

i) $y = \sin^{-1} 2x$

ii) $y = \tan^{-1} 2x$

(6 marks)

- d) Integrate the following function by parts

$$\int x \cos x dx$$

(5 marks)

- e) Find the volume of a solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$

(5 marks)

QUESTION TWO(20 MARKS)

- a) Find the integrals of the following by parts

i) $\int e^x \cos x dx$

(6 marks)

ii) $\int x^2 e^{2x} dx$

(6 marks)

- b) Find the integral of $\int \frac{dx}{4x^2 + 4x + 2}$

(5 marks)

- c) Find $\frac{\partial x}{\partial y}$ if $f(x,y) = y \sin xy$

(5 marks)

QUESTION THREE (20 MARKS)

a) Evaluate $I_n = \int \sin^n x dx$ (10 marks)

b) Approximate $\int_0^2 \frac{1}{x^2 + 1}$ with $n = 4$ using

i) Trapezoidal Rule (5 marks)

ii) Simpson's Rule (5 marks)

QUESTION FOUR (20 MARKS)

a) Evaluate $I_n = \int \frac{-2x + 4}{(x^2 + 1)(x - 1)^2}$ (12 marks)

b) Evaluate $\int 3x^2 \sqrt{x + 4} dx$ (8 marks)

QUESTION FIVE (20 MARKS)

a) Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -6$ (4 marks)

b) Find the length of the curve $y = \left(\frac{x}{2}\right)^{\frac{2}{3}}$ from $x = 0$ to $x = 2$ (5 marks)

c) Evaluate

i) $\int \cos^5 \theta d\theta$ (8 marks)

ii) $\int \cos^2 \theta d\theta$ (3 marks)