KABARAK   **UNIVERSITY**

UNIVERSITY EXAMINATIONS
2010/2011 ACADEMIC YEAR
FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS AND BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 113

COURSE TITLE: CALCULUS 1

STREAM: **Y1S1 & Y1S2**

DAY: **WEDNESDAY**

TIME: **2.00 – 4.00 P.M.**

DATE: **23/03/2011**

INSTRUCTIONS:

Attempt question **ONE** and any other **TWO** questions

PLEASE TURN OVER

Question One   (30mks)

(a)   i) Deduce the quotient rule of differentiation   **(4mks)**
       ii) Prove that the limit of the sequence.

$$Xn = \frac{2n}{3n-1} \quad \text{is} \quad \frac{2}{3} \quad \text{as } n \to \infty. \quad \text{Hence}$$

(4mks)

Find the values of N if   (i) $\in = 0.01$
                                            (ii) $\in = 0.001$
                                            (iii) $\in = 0.0001$

(4mks)

b) Given $y = \frac{1}{x}$ and x>0, use a sketch to show the two limiting values of the sequence.

(3 mks)

(c)   Evaluate the following limits.

(i) $\displaystyle \lim_{x \to 0} \frac{x^2 + x}{x}$ **(2mks)**

(ii) $\displaystyle \lim_{n \to \infty} \frac{n^2 + n}{n + 2}$ **(2mks)**

(iii) $\displaystyle \lim_{x \to 0} \frac{Cos - Cos3x}{x^2}$ **(2mks)**

(iv)   Find $\displaystyle \lim_{x \to \infty} \frac{(x-4)^{50}(2x+2)^{20}}{(6x^2 - 1)^{23}}$ **(2mks)**

(v) $\displaystyle \lim_{x \to 25} \frac{\sqrt{x} - 1}{x + 1}$ **(1mks)**

(d)   Find the derivatives $\frac{dy}{dx}$ of the following functions.

(i) $y = \sqrt{x^2 + 2x + 4}$ **(2mks)**

(ii) $y = x^2 (2x^2 + x + 3)^{-2}$ **(3mks)**

Question Two (20mks)

(a)   Show that:

    (i) $\dfrac{d}{dx}\,\mathrm{Sin}\ x = \mathrm{Cos}\ x$ **(4mks)**

    (ii) $\dfrac{d}{dx}\,\mathrm{Cos}\ x = \text{-}\mathrm{Sin}\ x$ **(4mks)**

(b)   Differentiate the following functions w.r.t x

    (i) $y = \dfrac{e^{-ax} + e^{ax}}{e^{ax}}$ **(4mks)**

    (ii) $y = \mathrm{Cos}^2\,(4x^2) + \mathrm{Sin}^3\,2x$ **(3mks)**

(c)   Evaluate the following Limit

$$\underset{x \to -\infty}{Lim}\left(1 + \frac{3}{x}\right)^{x+4}$$ **(5mks)**

Question Three (20mks)

a)   Find y^1 given $y = \dfrac{e^{ax} - e^{-ax}}{e^{ax} + e^{-ax}}$ **(9 mks)**

b)   Find $I = \displaystyle\int_{\sqrt{5}}^{2\sqrt{2}} \dfrac{x\,dx}{\sqrt{3x^2 + 1}}$ **(5 mks)**

c)   Find $\displaystyle\int_{\pi/18}^{\pi/9} \dfrac{dx}{\mathrm{Sin}^2\,3x}$ **(6 mks)**

Question Four (20mks)

(a)   Compute $\displaystyle\int\left(\dfrac{x^2}{5x^3 + 1}\right)dx$ **(5mks)**

(b)   Investigate the local extrema of the function.
       $f(x) = 2x^3 - 3x^2 - 12x + 5$ **(5mks)**

(c)   The gradient of a curve is $4x - 2$. Find the equation of the curve given x – axis is a tangent to the curve.   **(4mks)**

(d)   Using first principle method differentiate $\left(\dfrac{dy}{dx}\right)$

$y = \mathrm{Log}_a X$ **(6mks)**

Question Five (20mks)

(a)   An object starts from rest and gains an acceleration by $a(t) = 6t$. What is velocity and distance at $t = 7$ seconds?   **(6mks)**

(b)   Find y^1 given $y + 2xy - 11 + y^2 = 0$ **(4mks)**

(c)   Find the equations of the tangent and normal lines to the curve $y = 2x^2 + 4x - 3$ at the point where x=1   **(6mks)**

(d)   Evaluate $\displaystyle \lim_{x \to 0} \frac{\tan 6x}{8x}$ **(4mks)**