KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/2010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **MATH 313**

COURSE TITLE: **COMPLEX ANALYSIS**

STREAM: **SESSION V**

DAY: **TUESDAY**

TIME: **2.00 – 4.00 P.M.**

DATE: **06/04/2010**

INSTRUCTIONS:

Answer Question **ONE** and any other **TWO** Questions.

PLEASE TURN OVER

QUESTION ONE: (30 MARKS)

(a) Show the point $Z = 2 - 3i$ on an argard diagram and write it in polar form. [3 marks]

(b) Using the first principle technique find the derivatives of the following complex functions;

i) $f(z) = Cos\, z$ [5 marks]

ii) $f(z) = e^{-2z}$ [5 marks]

(c) Show that $f(z) = e^{z}$ is analytic everywhere on complex plane. [4 marks]

(d) If $f(z) = u + iv$ is analytic and $u = x^3 - 3xy^2$. Find V [3 marks]

(e) Verify Cauchy's theorem for the function $f(z) = z^3 + 2$ for circle $|z| = 2$ [5 marks]

(f) Evaluate $\oint_c \frac{z^6+1}{z^3(2z^2-5z+2)}\, dz$ Where C is the circle $|z| = 1$ [5 marks]

QUESTION TWO: (20 MARKS)

(a) Evaluate $\int_{(0,4)}^{(2,5)} (3x + y)dx + (2y - x)dy$ along

(i) The line $y = x^2 + 1$ [2 marks]

(ii) The straight line joining $(0, 1)$ and $(2, 5)$ [3 marks]

(iii) The straight lines from $(0, 1)$ to $(0, 5)$ and then from $(0, 1)$ to $(2, 5)$ [5 marks]

(b) Show that if $u + iv$ is analytic then $v - iu$ is also analytic. [3 marks]

(c) Evaluate $\int_0^{2\pi} \frac{d\theta}{(2+\cos\theta)^2}$ [7 marks]

QUESTION THREE: (20 MARKS)

(a) Derive the C – R Equations [15 marks]

(b) Prove that $\varphi = ln\{(x-1)^2 + (y-2)^2\}$ is harmonic in every region which does not include the point $(1, 2)$. [5 marks]

QUESTION FOUR: (20 MARKS)

(a) Show that $f(a) = \frac{1}{2\pi i} \oint \frac{f(z)}{z-a}\, dz$ [10 marks]

(b) Show that $f^1(a) = \frac{1}{2\pi i} \oint \frac{f(z)}{(z-a)^2}\, dz$ [10 marks]

QUESTION FIVE: (20 MARKS)

(a) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2(x^2+4)}$ [10 marks]

(b) Evaluate $\oint_0^{\infty} \frac{dx}{x^2+4}$ [10 marks]