KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

COURSE CODE: **MATH 313**

COURSE TITLE: **COMPLEX ANALYSIS**

STREAM: **SESSION III**

DAY: **MONDAY**

TIME: **2.00 – 4.00 P.M.**

DATE: **30/11/2009**

INSTRUCTIONS:

Attempt question **ONE** and any other **TWO** questions

PLEASE TURN OVER

QUESTION ONE

(a) Find the value of a and b if

$$\frac{a-bi}{1+2i} = 1 - 2i$$ **(3 marks)**

(b) Show that $|z| = |\overline{z}|$ for all z **(2 marks)**

(c) Given that $z = 2 + 3i$

Determine;

(i) $|z|$ (ii) Arg z **(2 marks)**

(d) Use De Moire Theorem to write $\sin 3\theta$ and $\cos 3\theta$ in terms of $\sin\theta$ and $\cos\theta$

(5 marks)

(e) Determine if and where the function z^2 is analytic **(7 marks)**

(f) Prove that $e^y \sin x$ is harmonic and find V such that $f(z) = u + iv$ is analytic.

(g) Evaluate the integral $\oint \frac{z^2+1}{(Z-1)^2}\,dz$ around the circle whose equation is $|z| = 2$.

(6 marks)

QUESTION TWO

(a) Determine the Taylor series expand of $\frac{1}{1+z}$ about the origin and state its radius of convergences. **(6 marks)**

(b) Use Residue theorem to Evaluate;

$$\oint_c \frac{dz}{z^2+1}$$ **(4 marks)**

(c) Derive the Canchy – Riemann conditions for analytic functions. **(10 marks)**

QUESTION THREE

(a) State Canchy's integral formula. **(2 marks)**

(b) Show that if $f(z)$ is analytic inside and on a simple closed curve c and a is any point inside C then $f(a) = \frac{1}{2\pi i} \oint_c \frac{f(Z)}{(Z-a)^2}\,dz$ **(10 marks)**

(c) Find the value of the integral $\oint \frac{z^2}{z^2 - 1}\, dz$ around the unit circle with centre at

(i) $z = 1$ (ii) $z = -1$ **(8 marks)**

QUESTION FOUR

(a) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region

(i) $|z| < 1$ **(4 marks)**

(ii) $|z| > 2$ **(4 marks)**

(b) State Green's Theorem **(7 marks)**

(c) Verify Green's Theorem in the plane for $\oint_C [(2xy - x^2)dx + (x + y^2)dy]$

where C is the closed curve of the region bounded by $y = x^2$ and $y^2 = x$. **(10 marks)**

QUESTION FIVE

(a) Use the first principle technique to find the derivative of the following complex functions

(i) $f(z) = \sin z$ **(4 marks)**

(ii) $f(z) = e^z$ **(4 marks)**

(b) Briefly explain the meaning of the following terms:

(i) Singularity **(2 marks)**

(ii) Removable singularity **(2 marks)**

(c) Express $\tan 4\theta$ in terms of $\tan \theta$ only using powers of complex numbers. **(4 marks)**

(d) If $f(z) = \frac{3z + 1}{(z-4)(z-1)}$, determine the poles and residues for $f(z)$. **(4 marks)**