KABARAK UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 313

COURSE TITLE: COMPLEX ANALYSIS

STREAM: **SESSION V**

DAY: **WEDNESDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **08/04/2009**

INSTRUCTIONS:

Answer question **ONE** and any other **TWO** questions

PLEASE TURN OVER

QUESTION ONE (30 Marks)

a) Briefly explain the following terms Pole. (2 marks)

b) Show that (+) = + . (4 marks)

c) By the first principle show that ( ) = − 2 has a derivative at every which is 3 − 2. (5 marks)

d) Evaluate the integral $\int$ ——— where is the circle | | = 4 traversed once in the counterclockwise direction. (4 marks)

e) Find the real part, imaginary part, modulus and argument of the complex number (4 + 2) −3 + $\sqrt{2}$. (5 marks)

f) Find the polar form of 2 + 2$\sqrt{3}$. (3 marks)

g) Solve the hyperbolic equation h − 5 h − 5 = 0. (5 Marks)

QUESTION TWO (20 Marks)

a) Derive Cauchy-Riemann conditions. (10 marks)

b) Define a harmonic function. (2 marks)

c) (i) Prove that the function (,) = − 3 + 3 − 3 + 1 satisfies Laplace's equation. (3 marks)

(ii) Determine the corresponding regular function + . (5 marks)

QUESTION THREE (20 Marks)

a) State Cauchy - Goursat theorem. (2 marks)

b) Verify Cauchy – Goursat theorem for the function ( ) = + 2 for the circle | | = 2. (6 marks)

c) Show that $\int$ ——— = − . (7 Marks)

d) Expand ( ) = in a Taylor series about =−. (5 marks)

QUESTION FOUR (20 Marks).

a) State and prove Cauchy Integral formula. (10 marks)

b) When is the circle $|\ | = 2$ described counter-clockwise, use Cauchy's integral formula to prove that $\int \frac{\quad}{\quad} = 0$ (4 Marks)

c) Determine the residue of the function $\frac{\quad}{(\quad)(\quad)}$ at all the poles. (6 marks)

QUESTION FIVE (20 Marks).

a) (i) State Laurent's theorem. (3 marks)

(ii) Expand in Laurent series the function $(\ ) = \frac{\quad}{(\quad)(\quad)}$ for $1 < |\ | < 5$. (8 marks)

b) Distinguish between conformal mapping and isogonal mapping. (4 Marks)

c) If $= (\ ) = \quad + \quad$ is analytic in a region , prove that $\frac{(\ ,\ )}{(\ ,\ )} = |\ (\ )|$. (5 marks)