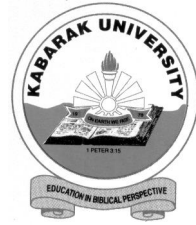


KABARAK



UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

**FOR THE DEGREE OF BACHELOR OF EDUCATION
SCIENCE**

COURSE CODE: MATH 313

COURSE TITLE: COMPLEX ANALYSIS I

STREAM: SESSION IV, V & VI

DAY: WEDNESDAY

TIME: 9.00 – 1.00 P.M.

DATE: 26/11/2008

INSTRUCTIONS TO CANDIDATES:

1. Answer Question **ONE** and any other **TWO** Questions

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) Express $f(z) = 2z^2 + 3z - 2$ in the form

$$f(z) = u + iv \quad (3 \text{ mks})$$

(b) Prove that $|\bar{z}_1, \bar{z}_2| = |z_1| |z_2|$ (4 mks)

(c) Expand $f(z) = \sin z$ in Taylor series about $z = \frac{\pi}{4}$ (5 mks)

(d) Find the Laurent series expansion for $f(z) = \frac{1}{z(z-1)}$ for $0 < |z| < 1$ (6 mks)

(e) Evaluate $\int_{(1,1)}^{(2,4)} (x^2 + ixy) dz$ along the curve $x = t, y = t^2$ (5 mks)

(f) Consider the function $f(z) = \frac{z^3 + 4}{(z^2 + 9)^2}$

(i) Find the residue of $f(z)$ at its poles (5 mks)

(ii) Evaluate $\int_c f(z) dz$ (2 mks)

QUESTION TWO (20 MARKS)

(a) State and prove the Cauchy – Riemann equation in the complex plane. (10 mks)

(b) Given $f(z) = 3z^2 + 6z + 4$. Verify Riemann equation for $f(z)$ (10 mks)

QUESTION THREE (20 MKS)

(a) If $f(z) = u(x, y) + iv(x, y)$ is an analytic function in some region of the z plane. Show that $f(z)$ is a harmonic function. **(10 mks)**

(b) Show that $u(x, y) = x^3y - y^3x$ is harmonic and find $V(x, y)$ so that $f(z) = u + iv$ is harmonic. **(10 mks)**

QUESTION FOUR (20 MKS)

(a) If $f(z)$ is analytic within and on a simple closed curve C and if A is any point within C , show that

$$f(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z-a} dz \quad \text{(10 mks)}$$

(b) When c is the circle described counter clockwise, use Cauchy's integral formula to prove that

(i) $\int_c \frac{z}{z^2 + 9} dz = 0$ **(5 mks)**

(ii) $\int_c \frac{z^2 + 4}{z - 1} dz = 10\pi$ **(5 mks)**

QUESTION FIVE (20 MARKS)

(a) Define singular point and isolated singular point. **(4 mks)**

(b) Prove that the series $z(1-z) + z^2(1-z) + z^3(1-z) + \dots$ converges for $|z| < 1$ **(7 mks)**

(c) Show that the series in question 5(b) above is absolutely convergent for $|z| < 1$ **(8 mks)**