KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 313

COURSE TITLE: **COMPLEX ANALYSIS I**

STREAM: **SESSION IV, V & VI**

DAY: **WEDNESDAY**

TIME: **9.00 – 1.00 P.M.**

DATE: **26/11/2008**

INSTRUCTIONS TO CANDIDATES:

1. Answer Question **ONE** and any other **TWO** Questions

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) Express $f(z) = 2z^2 + 3z - 2$ in the form

$f(z) = u + iv$ **(3 mks)**

(b) Prove that $|z, z_2| = |z|\ |z_2|$ **(4 mks)**

(c) Expand $f(z) = \text{Sin}\, z$ in Taylor series about $z = \frac{\pi}{4}$ **(5 mks)**

(d) Find the Laurent series expansion for $f(z) = \frac{1}{z(z-1)}$ for $0<|z|<1$ **(6 mks)**

(e) Evaluate $\int_{(1,1)}^{(2,4)} (x^2 + ixy)\ dz$ along the curve x = t, y = t^2 **(5 mks)**

(f) Consider the function $f(z) = \frac{z^3 + 4}{(z^2 + 9)^2}$

(i) Find the residue of $f(z)$ at its poles **(5 mks)**

(ii) Evaluate $\int_c f(z)\ dz$ **(2 mks)**

QUESTION TWO (20 MARKS)

(a) State and prove the Caunchy – Riemann equation in the complex plane. **(10 mks)**

(b) Given $f(z) = 3z^2 + 6z + 4$. Verify Riemann equation for $f(z)$ **(10 mks)**

QUESTION THREE (20 MKS)

(a) If $f(z) = u(x,y)+iv(x,y)$ is an analytic function in some region of the z plane. Show that $f(z)$ is a harmonic function. **(10 mks)**

(b) Show that $u(x,y)=x^3y-y^3x$ is harmonic and find V(x, y) so that

$f(z) = u+iv$ is harmonic. **(10 mks)**

QUESTION FOUR (20 MKS)

(a) If $f(z)$ is analytic within and on a simple closed curve C and if A is any point within C, show that

$$f(a) = \frac{1}{2\pi i}\oint_c \frac{f(z)}{z-a}\,dz$$ **(10 mks)**

(b) When c is the circle described counter clockwise, use Cauchy's integral formula to prove that

(i) $\int_c \frac{z}{z^2+9}\,dz=0$ **(5 mks)**

(ii) $\int_c \frac{z^2+4}{z-1}\,dz=10\pi$ **(5 mks)**

QUESTION FIVE (20 MARKS)

(a) Define singular point and isolated singular point. **(4 mks)**

(b) Prove that the series $z(1-z) + z^2(1-z) + z^3(1-z) +$ ---- converges for $|z| < 1$

(7 mks)

(c) Show that the series in question 5(b) above is absolutely convergent for $|z|<1$

(8 mks)