KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2010/2011 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **MATH 313**

COURSE TITLE: **COMPLEX ANALYSIS**

STREAM: **Y3 S1**

DAY: **SATURDAY**

TIME: **9.00 – 11.00 A.M**

DATE: **27/11/2010**

INSTRUCTIONS:

Attempt question **ONE** and any other **TWO** questions

PLEASE TURN OVER

QUESTION ONE

(a) Find the value of a and b if

$$\frac{a-bi}{1+2i} = 1 - 2i$$ **(3 marks)**

(b) Show that $|z| = |\overline{z}|$ for all z **(3 marks)**

(c) Given that $z = 2 + 3i$

Determine;

(i) $|z|$ (ii) Arg z **(4 marks)**

(d) Using first principle derive cosz (8mks

(e) Determine if the function z^2 is analytic **(7 marks)**

(f) Prove that $e^y \sin x$ is analytic

QUESTION TWO

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(a) Evaluate;

$$\oint_c \frac{z^6 + 1}{z^3(2z^2 - 5z + 2)} \, dz$$ **(10 marks)**

(c) Derive the Cauchy – Riemann conditions for analytic functions. **(10 marks)**

QUESTION THREE

(a) State Canchy's integral formula. **(2 marks)**

(b) Show that if $f(z)$ is analytic inside and on a simple closed curve c and a is any point inside C then $f(a) = \frac{1}{2\pi i} \oint_c \frac{f(Z)}{(Z-a)^2} \, dz$ **(10 marks)**

(c) Find the value of the integral $\oint \frac{z^2}{z^2 - 1} \, dz$ around the unit circle with centre at

(i) $z = 1$ (ii) $z = -1$ **(8 marks)**

QUESTION FOUR

(a) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region

(i) $|z| < 1$ **(5 marks)**

(ii) $|z| > 2$ **(5 marks)**

b(i)(Evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2\,(x^2+4)}$ **(10 marks)**

(ii) $\int_0^{2\pi} \frac{d\varphi}{(2+\cos\theta)^2}$

QUESTION FIVE

(a) Use the first principle technique to find the derivative of the following complex functions

(i) $f(z) = \sin z$ **(4 marks)**

(ii) $f(z) = e^z$ **(4 marks)**

(b) Briefly explain the meaning of the following terms:

(i) Singularity **(2 marks)**

(ii) Removable singularity **(2 marks)**

(c) Express $\tan 4\theta$ in terms of $\tan\theta$ only using powers of complex numbers. **(4 marks)**

(d) If $f(z) = \frac{3z+1}{(z-4)(z-1)}$, determine the poles and residues for $f(z)$. **(4 marks)**