KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MATH 313

COURSE TITLE: COMPLEX ANALYSIS

STREAM: SESSION V

DAY: FRIDAY

TIME: 2.00 – 4.00 P.M.

DATE: 13/08/2010

INSTRUCTIONS:

Attempt question **ONE** and any other **TWO** questions.

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) Show the point $z = 3e^{i45^0}$ on an argard diagram. **(4 marks)**

(b) Define continuity of a complex function at a point z_0. **(3 marks)**

(c) From first principles differentiate the complex functions at a specified point.

(i) $f(z) = e^z$ at $z = 0$ **(4 marks)**

(ii) $f(z) = \sin x$ at $z = 720°$ **(5 marks)**

(d) If $f(z) = u + iv$ is analytic function and $u = x^3 - 3xy^2$ Find V. **(4 marks)**

(e) Evaluate $\oint_C \frac{z^6 + 1}{z^3(2z^2 - 5z + 2)} dz$ **(6 marks)**

(f) If $f(z) = z^2 + 1$ show that $\oint f(z)dz = 0$ where C is the circle $|z - 2| = 5$ **(4 marks)**

QUESTION TWO (20 MARKS)

(a) Derive the C – R equations and hence verify whether $f(z) = \bar{z}$ is analytic or not. **(15 marks)**

(b) Given $u(xy) = x^3y - xy^3$. Show that u is harmonic and find the harmonic conjugate V. **(5 marks)**

QUESTION THREE (20 MARKS)

(a) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2 + 1)^2(x^2 + 4)}$ **(10 marks)**

(b) Evaluate $\int_0^{2\pi} \frac{d\varphi}{(2 + \cos\theta)^2}$ **(10 marks)**

QUESTION FOUR (20 MARKS)

(a) Evaluate $\int_{(0,4)}^{(2,5)} (3x + y)dx + (2y - x)dy$ along

(i) The curve $y = x^2 + 1$ **(4 marks)**

(ii) The straight line joining $(0, 1)$ and $(2, 5)$ **(4 marks)**

(iii) The straight lines from $(0, 1)$ to $(0, 5)$ and then from $(0, 1)$ to $(2, 5)$ **(5 marks)**

(b) Expand $ln\left(\frac{1+z}{1-z}\right)$ in a Taylor's sense about $z = 0$ and determine the region of convergence.

(4 marks)

(c) If $f(z) = \frac{3z+1}{(z-4)(z-1)}$ find the poles and residues at the poles for $f(z)$. **(3 marks)**

QUESTION FIVE (20 MARKS)

If $f(z)$ is analytic inside and on the boundary C of a simple connected region then show that

$$f(a) = \frac{1}{2\pi i}\int \frac{f(z)}{z-a}\,dz \quad \text{and} \quad f^{n}(a) = \frac{1}{2\pi i}\,n!\int \frac{f(z)}{(z-a)^{n+1}}$$ where $n = 1, 2, 3$ taking

a specific example of $n = 1$ and hence generalize. **(20 marks)**