

**KABARAK**



**UNIVERSITY**

**UNIVERSITY EXAMINATIONS  
2010/2011 ACADEMIC YEAR  
FOR THE DEGREE OF BACHELOR OF SCIENCE IN  
ECONOMICS**

**COURSE CODE: MATH 313**

**COURSE TITLE: COMPLEX ANALYSIS**

**STREAM: Y3S1**

**DAY: TUESDAY**

**TIME: 2.00 – 4.00 P.M.**

**DATE: 15/03/2011**

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**INSTRUCTIONS:**

Attempt question **ONE** and any other **TWO** questions

**PLEASE TURN OVER**

## QUESTION ONE

- (a) Find the value of a and b if

$$\frac{a + ib}{c + id} = 1 - 2i \quad (3 \text{ marks})$$

- (b) Show that  $|z| = |\bar{z}|$  for all  $z$

(3 marks)

- (c) Given that  $z = 2 + 3i$

Determine;

(i)  $|z|$  (ii)  $\text{Arg } z$  (4 marks)

- (d) Using first principle derive  $\cos z$

(8mks)

- (e) Determine if the function  $f(z) = \cos z$  is analytic

(7 marks)

- (f) Prove that  $\sin z$  is analytic

## QUESTION TWO

- (a) Evaluate;

$$\oint_C \frac{1}{z^2} dz \quad (10 \text{ marks})$$

- (c) Derive the Cauchy – Riemann conditions for analytic functions and laplace equations.

(10 marks)

## QUESTION THREE

- (a) State Cauchy's integral formula.

(2 marks)

- (b) Show that if  $f(z)$  is analytic inside and on a simple closed curve  $C$  and  $a$  is any point

inside  $C$  then  $f(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{(z-a)^2} dz$  (10 marks)

- (c) Find the value of the integral  $\oint_C \frac{1}{z} dz$  around the unit circle with centre at

(i)  $a = 1$  (ii)  $a = -1$  (8 marks)

### QUESTION FOUR

(a) Expand  $f(z) = \frac{1}{(z-1)(z-2)}$  in the region

(i)  $|z| < 1$  **(5 marks)**

(ii)  $|z| > 2$  **(5 marks)**

b(i) Evaluate  $\int_{-\infty}^{\infty} \frac{1}{(z-1)(z-2)} dz$  (10 marks)

(ii)  $\int \frac{1}{(z-1)} dz$

### QUESTION FIVE

(a) Use the first principle technique to find the derivative of the following complex functions

(i)  $f(z) = \sin z$  **(4 marks)**

(ii)  $f(z) = e^z$  **(4 marks)**

(b) Briefly explain the meaning of the following terms:

(i) Singularity **(2 marks)**

(ii) Removable singularity **(2 marks)**

(c) Express  $\tan 4\theta$  in terms of  $\tan \theta$  only using powers of complex numbers. **(4 marks)**

(d) If  $f(z) = \frac{1}{(z-1)(z-2)}$ , determine the poles and residues for  $f(z)$ . **(4 marks)**