

KABARAK



UNIVERSITY

**UNIVERSITY EXAMINATIONS
2010/2011 ACADEMIC YEAR
FOR THE DEGREE OF BACHELOR OF EDUCATION
SCIENCE**

COURSE CODE: MATH 313

COURSE TITLE: COMPLEX ANALYSIS

STREAM: SESSION VI & VII

DAY: WEDNESDAY

TIME: 9.00 – 11.00 A.M.

DATE: 13/04/2011

INSTRUCTIONS:

Attempt question ONE and any other TWO questions

PLEASE TURN OVER

QUESTION ONE

- (a) Show the point $z = 3 + 4i$ on argand diagram. (4 marks)
- (b) Define continuity of a complex functions at a point z_0 . (3 marks)
- (c) From first principles differentiate the complex functions at a specified point.
- $f(z) = z^2$ at $z = 0$ (4 marks)
 - $f(z) = \sin z$ at $z = 720^\circ$ (5 marks)
- (d) If $f(z) = u + iv$ is analytic function and $u = x^2 - y^2$ Find v . (4 marks)
- (e) Evaluate $\oint_C \frac{1}{z} dz$ where C is the circle $|z| = 1$ (6 marks)
- (f) If $f(z) = u + iv$ show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ if $|z - 2| = 5$. (4 marks)

QUESTION TWO (20 MARKS)

- (a) Derive the C – R equations and hence verify whether $f(z) = z^2$ is analytic or not. (15 marks)
- (b) Given $u(x, y) = x^2 - y^2$. Show that u is harmonic and find the harmonic conjugate v . (5 marks)

QUESTION THREE (20 MARKS)

- (a) Evaluate $\int_0^\infty \frac{1}{(x^2 + 1)^2} dx$ (10 marks)
- (b) Evaluate $\int \frac{1}{z^2} dz$ (10 marks)

QUESTION FOUR (20 MARKS)

- (a) Evaluate $\int_C (3x + y) dx + (2x - y) dy$ along
- The curve $y = x^2 + 1$ (4 marks)
 - The straight line joining (0,1) and (2,5) (4 marks)
 - The straight line joining (0,1) to (0,5) and then from (0,1) to (2,5) (5 marks)

(b) Expand $\frac{1}{z-4}$ in a Taylor's series about $z = 0$ and determine the region of convergence. (4mks)

(c) $f(z) = \frac{3z+1}{(z-4)(z-1)}$ find the poles and residues at the poles for $f(z)$ (3marks)

QUESTION FIVE (20MARKS)

If $f(z)$ is analytic inside and on the boundary C of a simple connected region then show that $f^{(n)}(z) = \frac{n!}{2\pi i} \int_C \frac{f(\zeta)}{(\zeta-z)^{n+1}} d\zeta$ and $f^{(n)}(z) = \frac{n!}{2\pi i} \int_C \frac{f(\zeta)}{(\zeta-z)^{n+1}} d\zeta$ where $n = 1, 2, 3$ taking a specific example of $f(z) = \frac{1}{z}$ and hence generalize. (20 marks)