KABARAK UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN COMPUTER SCIENCE

COURSE CODE: COMP 122

COURSE TITLE: DISCRETE STRUCTURE

STREAM: Y1S2

DAY: MONDAY

TIME: 8.30 - 10.30 A.M.

DATE: 15/12/2008

INSTRUCTIONS:

Note: - Part-A is compulsory, has 30 marks and from Part-B, You can attempt any two questions. Each question has 20 marks.

PLEASE TURN OVER

Part-A

QUESTION 1

a) Rewrite the following statements using set notation:

i) the element 1 is not a member of A

ii) A is a subset of C

iii) F contains all the elements of G. Marks 1.5

b) State the following:

i) The principle of extension and

ii) The principle of abstraction. Mark 1

c) List the elements of the following sets; here N={1,2,3……}.

i) $A = \{ x: x \in N, 3<x<12\}$

ii) $B = \{ x: x \in N, x \text{ is even}, x<15\}$

iii) $C = \{ x: x \in N, 4+x = 3\}$

iv) $A = \{ x: x \in N, x^2 + 1 = 10\}$

v) $B = \{ x: x \in N, x \text{ is odd}, -5 < x < 5\}$ Marks 2.5

d) Let $A = \{ x: 3x = 6\}$.Explain does $A = 2$? Marks 2

e) Which of these sets are equal: {r,s,t}, {t,s,r}, {s,r,t}, {t,r,s}? Marks 2

f) Consider the sets:

$\{4,2\}$, $\{x: x^2 -6x + 8=0\}$, $\{x: x\in N, x \text{ is even}, 1<x<5\}$.

Which of them are equal to $B = \{2,4\}$? Marks 2

g) Describe a situation where the universal set U may be empty. Marks 3

h) Explain the difference between $A\subseteq B$ and $A\subset B$. Marks 2

i) Show that $A = \{2,3,4,5\}$is not a subset of $B= \{x: x\in N, x \text{ is even}\}$. Marks 2

j) Suppose $A = \{1,2\}$. Find

i) A^2

ii) A^3 Marks 2

k) A class consists of seven men and five women. Find the number m of committees of five that can be selected from the class. Marks 2

l) Verify that the proposition $p\vee\neg(p\wedge q)$ is a tautology. Marks 4

m) Determine the power set P(A) of A={a,b,c,d} Marks 4

Question 2

a) Consider the following sets:

I. X= {x: x is an integer, x>1}

II. Y= {y: y is an positive integer, divisible by 2}

III. Z= {z: z is an even number , greater than 2}

Which of them are subset of w = {2,4,6…….}? Marks 3

b) Suppose that A= {1,2,3,4}, B= {2,3,4,5,6,7}, C= {3,4}, D= {4,5,6}and E= {3}

1. which of the five sets can equal X if $X \subseteq A$ and $X \subseteq B$?
2. which of the five sets can equal to X if $X \not\subset D$ and $X \subseteq C$?
3. find the smallest set M which contains all five sets.
4. Find the largest set N which is a subset of all the five set. Marks 4

c) Draw a venn diagram of sets A, B, C where A and B have elements in common, B and C have elements in common, but A and C are disjoint. Marks 2

d) Draw a venn diagram of sets A, B, C where $A \subseteq B$, sets A and C are disjoint, but B and C have elements in common. Marks 2

e) Suppose U={1,2,3,……..8,9},A= {1,2,3,4}, B={2,4,6,8}, and C={3,4,5,6}. Find

(i) $(A \cup B) \cup C$ and (ii) $A \cup (B \cup C)$ Marks 4

f) Determine which of the following sets are finite.

(i) A={seasons in the year}

(ii) B= {state in the union}

(iii) C={+ve integers less than 1} Marks 3

g) Let R be the relation on A={1,2,3,4} defined by:

R = {(1,1),(3,1),(3,4),(4,2),(4,3)}

Find the composition $R^2 = R \circ R$ from the relation Marks 2

Question 3

a) Suppose U={1,2,3,……..8,9},A= {1,2,3,4}, B={2,4,6,8}, and C={3,4,5,6}.

Find (i) A^c

(ii) B^c

(iii) A\B

(iv) B\B Marks 4

b) Suppose U={1,2,3,……..8,9},A= {1,2,3,4}, B={2,4,6,8}, and C={3,4,5,6}.

Find (i) $(A \cap B) \setminus C$

(ii) $(A \setminus B)^c$ Marks 2

c) Prove the commutative laws: (i) $A \cup B = B \cup A$ and (ii) $A \cap B = B \cap A$

Marks 2

d) Write the dual of each set equation:

(i) $(A \cup B \cup C)^c = (A \cup C)^c \cap (A \cup B)^c$

(ii) $(A \cup U) \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ Marks 2

e) Prove the absorption laws: $A \cup (A \cap B) = A$ Marks 4

f) Translate each of the following statements in to a venn diagram.

(i) all students are lazy

(ii) some students are lazy. Marks 2

g) Given $A = \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 0 \\ -6 & 7 \end{pmatrix}$. Find $(AB)^T$ Marks 2

h) Find the number of distinct permutations that can be formed from all the letters of each word

I. THEM; and

II. THAT Marks 2

Question 4

a) Find the number of elements in the finite set:

(i) A={2,4,6,8,10}

(ii) $B=\{x: x^2 = 4\}$

(iii) $C=\{x: x > x + 2\}$ Marks 3

b) One hundred students were asked whether they had taken courses in any of the three areas, sociology, anthropology, and history. The result were:

45 had taken sociology

38 had taken anthropology

21 had taken history

18 had taken sociology and anthropology

9 had taken sociology and history

5 had taken history and anthropology and

23 had taken no courses in any of the area.

(i) Draw a venn diagram that will show the results of the survey. Marks 4

(ii) Determine the number k of students who had taken classes in exactly (1) one of the areas, and (2) two of the areas. Marks 2

c) Let A = {1,2,3} and B = {a,b}. find $A \times B$ Marks 2

d) Given A = {1,2}, B ={x,y,z}, and C = {3,4}

Find $A \times B \times C$ and $n(A \times B \times C)$ by the help of tree diagram. Marks 3

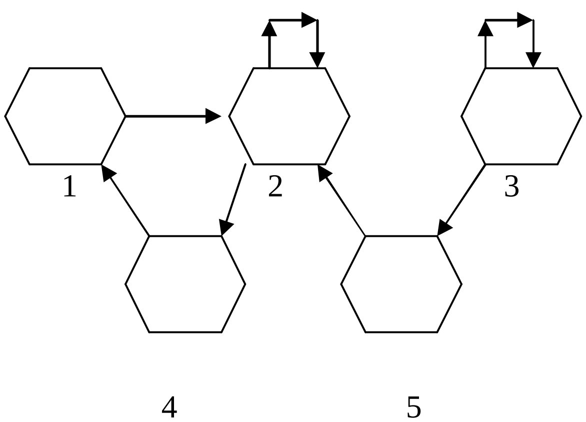
e) Let R be the relation from A = {1,2,3,4}to B = {x,y,z}defined by

R = {(1,y),(1,z),(3,y),(4,x),(4,z)}

(i) determine the domain and range of R

(ii) find the inverse relation R^{-1} of R. Marks 2

f) Let R be the relation on A = {1,2,3,4,5} described by the directed graph in the fig. write R as a set of ordered pairs. Marks 2



g) Consider a relation S from X = {1,2,3} to Y = {a,b,c,d}whose matrix representation is

$$M = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ 2 & \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

Marks 2

Question 5

a) Describe the " arrow diagram" of a relation R from a finite set A to a finite set B. Illustrate using the relation R from set A ={1,2,3,4} to set B = {x,y,z}defined by

R = {(1,y),(1,z),(3,y),(4,x),(4,z)} Marks 2

b) Consider the following three relations on the set A= {1,2,3}:

R= {(1,1),(1,2),(1,3),(3,3)}

S = {(1,1),(1,2),(2,1),(2,2),(3,3)}

T = AXA

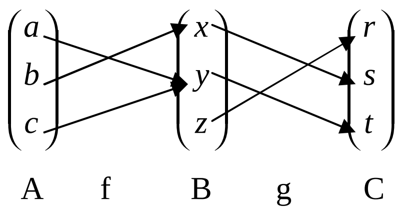
(i) Determine which of the relations are reflective.

(ii) Determine which of the relations are symmetric.

(iii) Determine which of the relations are transitive. Marks 3

c) Functions f: A$\rightarrow$B, g: B$\rightarrow$C

Find the composition function h o g Marks 3



d) Prove the associative law: $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ Marks 4

e) Use a K-map to find the prime implicants and minimal form for each of the following complete sum-of-products Boolean expressions.

E_1=xyz + xyz' + xy'z + x'yz + x'y'z Marks 3

f) Design a three-input minimal AND-OR circuit L that will have the following truth table:

T= [A=00001111, B= 00110011, C= 01010101, L= 11001101

Marks 3

g) A student is to answer eight out of ten questions on an exam. Find the number m of ways that the student can choose the eight questions. Marks 2