## COURSE CODE: COMP 122

## COURSE TITLE: DISCRETE STRUCTURE

## STREAM: Y1S2

DAY: WEDNESDAY
TIME:
9.00-11.00 A.M.

DATE:
05/08/2009

## INSTRUCTIONS:

1. There are 5 questions in the paper answer question one, which is compulsory and any other two questions.
2. Be Brief and precise.
3. Question one carries 30 marks

## PLEASE TURN OVER

## PART ONE

## Question One (Marks 30) Compulsory

a)

If $A=\{x: x$ is an English alphabet $\}$
$B=\{x: x$ is a vowel in English alphabet $\}$
Find $A \cap B$
Marks 2
b)

If $A=\{a, b, c, d, e, f, g\}$ and
$B=\{a, b, c, d, h, I, j\}$
Find (i) $A-B$
(ii) $\mathrm{B}-\mathrm{A}$
Marks 2
c)

Suppose U=\{1,2,3, ..........10\}
$A=\{1,3,5,7,9\}$
Find $\mathrm{A}^{\mathrm{c}}$
Marks 2
d)

Let $A$ and $B$ be two sets. Then Prove that $(A \cap B)^{c}=A^{c} \cup B^{c}$
e)

List the elements of the following sets; here $\mathrm{N}=\{1,2,3 \ldots \ldots\}$.
i) $\quad A=\{x: x \in N, 3<x<12\}$
ii) $\quad B=\{x: x \in N, x$ is even, $x<15\}$
iii) $\quad C=\{x: x \in N, 4+x=3\}$
iv) $\quad A=\left\{x: x \in N, x^{2}+1=10\right\}$
v) $\quad B=\{x: x \in N$, $x$ is odd, $-5<x<5\} \quad$ Marks 5
f)

Let $X=\{1,2,3,4,5,6\}$
Let $R$ and $S$ be relations from $X$ to $X$ as
$R=\{(x, y) /(x+y)$ is a multiple of 2$\}$
$S=\{(x, y) /(x+y)$ is a multiple of 3$\}$
Find $R \cup S$ and $R \cap S$
Marks 2
g)

Let $A\{a, b, c, d\}, B=\{a, e, f, d\}$ and $R=\{(a, e),(a, f),(b, e),(c, f),(b, d),(d, d),(d, a)\}$. Draw the relation graph. Marks 4
h)

Let $A=\{1,2,3,4\}, B=\{1,3,9,10\}, C\{5,6,7,8\}$
$R=\{(1,1),(1,3),(2,9),(2,10),(3,3),(4,10)\}$
$\mathrm{S}=\{(1,5),(3,7),(9,7),(10,8)\}$
Find $\operatorname{RoS}$ and its relation graph by the help of corresponding matrix.
i)

Convert the following POS to a SOP:
$(A+B C)(A+B D)$
Marks 2

## PART TWO

## Question Two (Marks 20)

a)

Let $\mathrm{f}: \mathrm{Z} \rightarrow \mathrm{Z}$ be a function defined by $\mathrm{f}(\mathrm{x})=2 \mathrm{x}+3$ : let $\mathrm{g}: \mathrm{Z} \rightarrow \mathrm{Z}$ be a function defined by $\mathrm{g}(\mathrm{x})=3 \mathrm{x}+2$.
Find (i) fog
(ii) g of

Marks 4
b)

In how many ways can the letters of word ENGINEERING be arranged
(i) Without changing the order of the consonants.
(ii) Without changing the relative positions of the vowels and the consonants.

Marks 4
c)

If $A=\left[\begin{array}{ccc}2 & -1 & 0 \\ 0 & -2 & 1 \\ 1 & 0 & 1\end{array}\right] ; B=\left[\begin{array}{ccc}-2 & 1 & -1 \\ 1 & 2 & -2 \\ 2 & -1 & -4\end{array}\right]$ and $C=\left[\begin{array}{ccc}1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3\end{array}\right]$
Evaluate 2A+3B-4C
Marks 3
d)

Prove the Theorem (Idempotent Laws) A $+\mathrm{A}=\mathrm{A}$ and $\mathrm{AA}=\mathrm{A} \quad$ Marks 2
e)

Prove that $\mathrm{A}^{3}-4 \mathrm{~A}^{2}-3 \mathrm{~A}+11 I=0$ when $\mathrm{A}=\left[\begin{array}{ccc}1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3\end{array}\right]$ and $I$ is the unit matrix of order 3
Marks 4
f)

Let $p$ be "Ravi is rich" and $q$ be "Ravi is happy". Write each of the following in symbolic form
(i) Ravi is poor but happy.
(ii) Ravi is neither rich nor happy.
(iii) Ravi is rich and unhappy. Marks 3

## Question Three (Marks 20)

a)

If $A=\left[\begin{array}{lll}1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9\end{array}\right]$; and $B=\left[\begin{array}{lll}2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1\end{array}\right]$
Find $A^{-1}, B^{-1}$ and hence show that $(A B)^{-1}=B^{-1} A^{-1}$
Marks 6
b)

Construct the truth table for the statement formula $\sim \mathrm{p} \wedge \mathrm{q} \quad$ Marks 2
c)

Construct the truth table for $(p \vee q) \vee \sim p$
Marks 3
d)

Construct the truth table for $(\mathrm{p} \rightarrow \mathrm{q}) \rightarrow(\mathrm{p} \wedge \mathrm{q})$
Marks 3
e)

Show that two propositions $\mathrm{p} \rightarrow \mathrm{q}$ and $\sim \mathrm{p} \vee \mathrm{q}$ are logically equivalent.
ie $\mathrm{p} \rightarrow \mathrm{q} \Leftrightarrow \sim \mathrm{p} \vee \mathrm{q} \quad$ Marks 4
f)

Prove that $\mathrm{A}(\mathrm{BC})=(\mathrm{A} \mathrm{B})(\mathrm{AC}) . \quad$ Marks 2

Question Four (Marks 20)
a) Show that $(\mathrm{p} \wedge \mathrm{q}) \longrightarrow(\mathrm{p} \vee \mathrm{q})$ is a tautology. Marks 4
b)

Draw logical networks for $(\mathrm{a} \cdot \bar{b})+(\bar{a} \cdot \mathrm{~b})$ Marks 3
c)

Write the adjacency matrix of the following graphs.

d) Write the truth table for the biconditional.

## Marks 2

e)

Let the universal set be $U-\{1,2,3$..
$A=\{2,3,7,9\} ; B=\{1,4,6,7,10\}$ and $C=\{3,5,7,9\}$
Draw venn diagram and find
(i) $\mathrm{A} \cup \mathrm{B}$
(ii) $\mathrm{A} \cap \mathrm{C}$
(iii) $\mathrm{B} \cap \bar{C}$
(iv) $(\mathrm{A} \cap \bar{B}) \cup \mathrm{C}$
(v) $\overline{B \cup C} \cap \mathrm{C}$

Marks 6

## Question Five (Marks 20)

a)

Use a k-map to simplify the Boolean expression
$\mathrm{xyz}+\mathrm{xy} \bar{z}+\mathrm{x} \bar{y} \bar{z}+\bar{x} \bar{y} \mathrm{z}+\bar{x} \bar{y} \bar{z}$
Marks 4
b)

How many anagrams (permulations of letters) are there of the word ANSWER
Marks 2
c)

A part of a market research survey, you are shown a list of 20 types of chocolate bar, and asked to list your five favorite bars in order of preference. How many different responses to this question are there?

Marks 2
d)

A task group consisting of three women and three men in to be chosen from the members of an Organisation consisting of 23 women and 18 men. In how many ways can this be done?

## Marks 2

e)

Draw the graph with the following adjacency matrix.
$\left.\begin{array}{c} \\ \boldsymbol{a} \\ \boldsymbol{a} \\ \boldsymbol{c} \\ \boldsymbol{d}\end{array} \begin{array}{cccc}\boldsymbol{a} & b & c & \boldsymbol{d} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 1 & 1\end{array}\right]$

## Marks 2

f)

Design a three-input minimal AND-OR circuit L that will have the following truth table:
$\mathrm{T}=[\mathrm{A}=00001111, \mathrm{~B}=00110011, \mathrm{C}=01010101, \mathrm{~L}=11001101$
Marks 3
g)
d) Given $\mathrm{A}=\{1,2\}, \mathrm{B}=\{\mathrm{x}, \mathrm{y}, \mathrm{z}\}$, and $\mathrm{C}=\{3,4\}$

Find $\mathrm{A} \times \mathrm{B} \times \mathrm{C}$ and $\mathrm{n}(\mathrm{A} \times \mathrm{B} \times \mathrm{C})$ by the help of tree diagram.
Marks 5

