KABARAK UNIVERSITY

EXAMINATIONS

2009/2010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF COMPUTER SCIENCE

COURSE CODE: COMP 122

COURSE TITLE: DISCRETE STRUCTURE

STREAM: Y1S2

DAY: WEDNESDAY

TIME: 9.00 – 11.00 A.M.

DATE: 09/12/2009

INSTRUCTIONS:

- Answer **ALL** questions in **SECTION A** and **ANY TWO** in **SECTION B**.
- Indicate question numbers clearly at the top of each page.
- Observe further instructions on the answer booklet.

PLEASE TURN OVER

Part-A

QUESTION 1

a) Rewrite the following statements using set notation:

(i) the element 1 is not a member of A

(ii) A is a subset of B — Marks 2

b) State the following:

(i) The principle of extension and

(ii) The principle of abstraction. — Marks 2

c) List the elements of the following sets; here N={1,2,3……}.

(i) $A = \{ x: x \in N, 6<x<12\}$

(ii) $B = \{ x: x \in N, x \text{ is even}, x<11\}$

(iii) $C = \{ x: x \in N, 4+x = 3\}$

(iv) $A = \{ x: x \in N, x^2 + 1 = 10\}$

(v) $B = \{ x: x \in N, x \text{ is odd}, -5 < x < 6\}$ — Marks 5

d) Let $X = \{x: 3x = 6\}$.Explain does $X = 2$? — Marks 2

e) Which of these sets are equal: {r,s,t}, {t,s,r}, {s,r,t}, {t,r,s}? — Marks 2

f) Consider the sets:
$\{4, 2\}$, $\{x: x^2 -6x + 8=0\}$, $\{x: x\in N, x \text{ is even}, 1<x<5\}$.
Which of them are equal to $B = \{2, 4\}$? — Marks 2

g) Draw the K-Map of the following expression. $Z = f(A,B,C) = \overline{A}\overline{B}\overline{C} + \overline{A}B + AB\overline{C} + AC$
Marks 3

h) Explain the difference between $A \subseteq B$ and $A \subset B$. — Marks 2

i) Show that $A = \{2, 3, 4, 5\}$ is not a subset of $B= \{x: x\in N, x \text{ is even}\}$. — Marks 2

j) Suppose $A = \{1,2\}$. Find

(i) A^2

(ii) A^3 — Marks 2

k) A class consists of seven men and five women. Find the number m of committees of five that can be selected from the class. — Marks 2

l) Determine the power set P(A) of A={a, b, c, d, e} — Marks 4

Question 2

a) Consider the following sets:

(I) $X= \{x: x \text{ is an integer}, x>1\}$

(II) $Y= \{y: y \text{ is an positive integer, divisible by } 2\}$

(III) Z= {z: z is an even number , greater than 2}
Which of them are subset of w = {2, 4, 6…….}? Marks 3

b) Suppose that A= {1,2,3,4}, B= {2,3,4,5,6,7}, C= {3,4}, D= {4,5,6}and E= {3}

(i) Which of the five sets can equal X if $X \subseteq A$ and $X \subseteq B$?
(ii) Which of the five sets can equal to X if $X \not\subset D$ and $X \subseteq C$?
(iii) Find the smallest set M which contains all five sets.
(iv) Find the largest set N which is a subset of all the five set.

Marks 4

c) Draw a venn diagram of sets A, B, C where A and B have elements in common, B and C have elements in common, but A and C are disjoint. Marks 2

d) Draw a venn diagram of sets A, B, C where $A \subseteq B$, sets A and C are disjoint, but B and C have elements in common. Marks 2

e) Suppose U={1,2,3,……..8,9},A= {1,2,3,4}, B={2,4,6,8}, and C={3,4,5,6}. Find

(i) $(A \cup B) \cup C$ and
(ii) $A \cup (B \cup C)$ Marks 4

f) Determine which of the following sets are finite.

(i) A={seasons in the year}
(ii) B= {state in the union}
(iii) C={+ve integers less than 1} Marks 3

g) Let R be the relation on A={1,2,3,4} defined by:
R = {(1,1),(3,1),(3,4),(4,2),(4,3)}
Find the composition $R^2 = R \circ R$ from the relation Marks 2

QUESTION 3

a) Suppose U={1,2,3,……..8,9},A= {1,2,3,4}, B={2,4,6,8}, and C={3,4,5,6}.

(i) Find A^c
(ii) A\B
(iii) B\B Marks 4

b) Suppose U={1,2,3,……..8,9},A= {1,2,3,4}, B={2,4,6,8}, and C={3,4,5,6}.
Find (i) $(A \cap B) \setminus C$
(ii) $(A \setminus B)^c$ Marks 2

c) Prove the commutative laws: (i) $A \cup B = B \cup A$ and (ii) $A \cap B = B \cap A$

Marks 2

d) Write the dual of each set equation:

(i) $(A \cup B \cup C)^c = (A \cup C)^c \cap (A \cup B)^c$
(ii) $(A \cup U) \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ Marks 2

e) Prove the absorption laws: $A \cup (A \cap B) = A$ Marks 4

f) Translate each of the following statements in to a venn diagram.

(i) all students are lazy

(ii) some students are lazy. Marks 2

g) Given A = $\begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}$ and B= $\begin{pmatrix} 5 & 0 \\ -6 & 7 \end{pmatrix}$. Find $(AB)^T$ Marks 2

h) Consider the expression $Z = f(A,B) = \bar{A}\bar{B} + A\,\bar{B} + \bar{A}B$ .Plot the Karnaugh map and find the minimal form of Z. Marks 2

QUESTION 4

a) Find the number of elements in the finite set:

(i) A={2,4,6,8,10}

(ii) $B=\{x: x^2 = 4\}$

(iii) $C=\{x: x > x + 2\}$ Marks 3

b) One hundred students were asked whether they had taken courses in any of the three areas, sociology, anthropology, and history. The result were:

45 had taken sociology

38 had taken anthropology

21 had taken history

18 had taken sociology and anthropology

9 had taken sociology and history

5 had taken history and anthropology and

23 had taken no courses in any of the area.

(i) Draw a venn diagram that will show the results of the survey. Marks 4

(ii) Determine the number k of students who had taken classes in exactly (1) one of the areas, and (2) two of the areas. Marks 2

c) Let A = {1, 2, 3} and B = {a, b}. find $A \times B$ Marks 2

d) Given A = {1,2}, B ={x, y, z}, and C = {3,4}

Find $A\times B\times C$ and $n(A\times B\times C)$ by the help of tree diagram. Marks 3

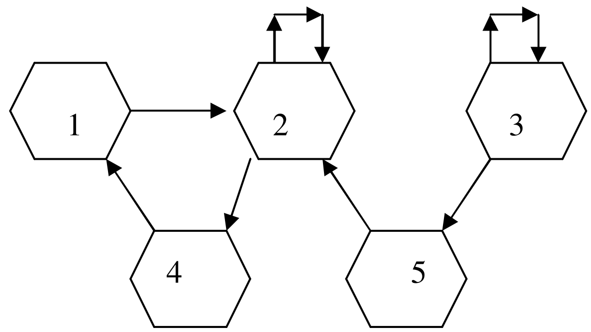
e) Let R be the relation from A = {1, 2, 3, 4} to B = {x, y, z}defined by

R = {(1,y),(1,z),(3,y),(4,x),(4,z)}

(i) Determine the domain and range of R

(ii) Find the inverse relation R^{-1} of R. Marks 2

f) Let R be the relation on A = {1,2,3,4,5} described by the directed graph in the fig. write R as a set of ordered pairs. Marks 2



g) Draw the k-map of the expression: $Z = f(A,B) = A\overline{B} + AB$ Marks 2

QUESTION 5

a) Describe the "arrow diagram" of a relation R from a finite set A to a finite set B. Illustrate using the relation R from set A ={1,2,3,4} to set B = {x, y, z}defined by R = {(1, y),(1,z),(3,y),(4,x),(4,z)} Marks 2

b) Consider the following three relations on the set A= {1,2,3}:

R= {(1,1),(1,2),(1,3),(3,3)}

S = {(1,1),(1,2),(2,1),(2,2),(3,3)}

T = AXA

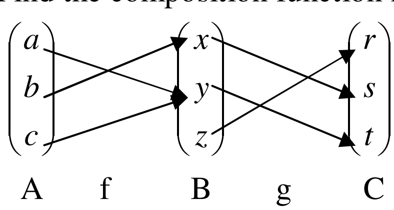
(i) Determine which of the relations are reflective.

(ii) Determine which of the relations are symmetric.

(iii) Determine which of the relations are transitive. Marks 3

c) Functions f: $A \rightarrow B$, g: $B \rightarrow C$

Find the composition function h o g Marks 3



d) Prove the associative law: $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ Marks 4

e) Use a K-map to find the prime implicants and minimal form for each of the following complete sum-of-products Boolean expressions.
E_1=xyz + xyz' + xy'z + x'yz + x'y'z Marks 3

f) Design a three-input minimal AND-OR circuit L that will have the following truth table:
T= [A=00001111, B= 00110011, C= 01010101, L= 11001101 Marks 3

g) Using algebraic method, simplify $Z = A\overline{B} + AB$ Marks 2