

KABARAK



UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

**FOR THE DEGREE OF BACHELOR OF SCIENCE IN
COMPUTER SCIENCE**

COURSE CODE: COMP 122

COURSE TITLE: DISCRETE STRUCTURE

STREAM: Y1S2

DAY: FRIDAY

TIME: 9.00 – 11.00 A.M.

DATE: 20/03/2009

INSTRUCTIONS:

1. There are 5 questions in the paper answer question one, which is compulsory and any other two questions.
2. Be Brief and precise.
3. Question one carries 30 marks

PLEASE TURN OVER

PART ONE
Question One (Marks 30) Compulsory

a) Rewrite the following statements using set notation

- (i) the element 1 is not a member of A
- (ii) A is subset of C
- (iii) F contains all the statement of G
- (iv) E and F contain the same elements.

Marks 4

b) List the elements of the following sets; here $Z = \{\text{integers}\}$

- (i) $A = \{x : x \in Z, 3 < x < 9\}$
- (ii) $B = \{x : x \in Z, x^2 + 1 = 10\}$
- (iii) $C = x + Z, x \text{ is odd}, -5 < x < 5$

Marks 3

c) Prove: If A is a subset of the null set ϕ ; then $A = \phi$

Marks 2

d) Draw a venn diagram of set A, B, C where $A \subseteq B$, set A and B are disjoint, but B and C have elements in common.

Marks 2

e) Find x and y if $(x-3y, 5) = (7, x-y)$

Marks 2

f) Let $A = \{1, 2, 3\}$ and $B = \{a, b\}$, Find $A \times B$

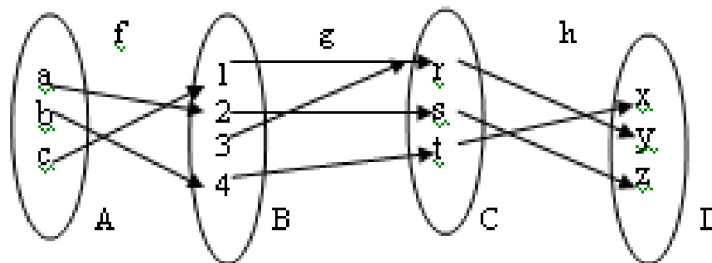
Marks 2

g) Given $A = \{1, 2\}$, $B = \{x, y, z\}$, and $C = \{3, 4\}$, Find $A \times B \times C$ and $n(A \times B \times C)$

Marks 4

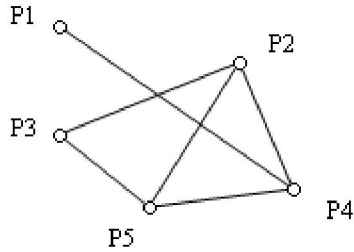
h) Determine whether each of the functions f, g and h in fig is invertible, and if it is, find its inverse.

Marks 2



i) Describe formally the graph shown in figure

Marks 2



- j) Simplify $\frac{(x - r + 1)!}{(x - r - 1)!}$ Marks 2
- k) Compute $\binom{9}{7}$ Mark 1
- l) Find the numbers of ways that a party of seven persons can arrange them selves in arrow of seven chairs Marks 2
- m) There are twelve students who are eligible to attend the national student association annual meeting. Find the number m of ways a delegation of four students can be selected from the twelve eligible students. Marks 2

PART TWO
Question two (Marks 20)

- a) Let $X = \{\text{red, blue}\}$
 $Y = \{\text{blue, green, orange}\}$
 $Z = \{\text{red, blue, white}\}$
 $U = \{\text{red, yellow, blue, green, orange, purple, black, white}\}$
 Find (i) $X \setminus Y$
 (ii) $(X \cap Y \cap Z)$ Marks 2
- b) Prove the absorption law: $A \cup (A \cap B) = A$ Marks 2
- c) In a survey of 60 people, it was found that 25 read Newsweek magazine, 26 read Time and 26 read Fortune. Also 9 read both Newsweek and Fortune, 11 read both Newsweek and Time, 8 read both Time and Fortune, and 8 read no magazine at all.
 (i) Find the number of people who read the entire three magazine.
 (ii) Draw the VENN diagram of all.
 (iii) Determine the number of people who read exactly one magazine. Marks 5
- d) Find the power $P(A)$ of $A = \{1, 2, 3, 4, 5\}$ Marks 2

e) Prove $(A \times B) \cap (A \times C) = A \times (A \cap C)$ Marks 2

f) Let R be the relation from $A = \{1, 2, 3, 4\}$ to $B = \{x, y, z\}$ defined by

$$R = \{(1, y), (1, z), (3, y), (4, x), (4, z)\}$$

(i) determine the domain and range of R

(ii) find the inverse of relation R^{-1} of R Mark 1

g) Describe the "arrow diagram" of a relation R from a finite set A to a finite set B. Illustrate using the relation R from $A = \{1, 2, 3, 4\}$ to set $B = \{x, y, z\}$ defined by

$$R = \{(1, y), (1, z), (3, y), (4, x), (4, z)\}$$

Marks 2

h) Describe the "directed graph" of a relation R on a set A. illustrate using the relation R on $A = \{1, 2, 3, 4\}$ defined by

$$R = \{(1, 2), (2, 2), (2, 4), (3, 2), (3, 4), (4, 1), (4, 3)\}$$

Marks 2

i) Let R be the relation on $A = \{2, 3, 4, 6, 9\}$ defined by "x is relatively prime to y" i.e. the only +ve divisor of x and y is 1

(i) write R as a set of ordered pairs.

(ii) Find the matrix M representing the relation R Mark 1

j) Let $u = (2, 3, -4)$ and $v = (1, -5, 8)$

Find (i) $u + v$

(ii) $5u$ Mark 1

Question three (Marks 20)

a) Find $2A - 3B$, where $A = \begin{pmatrix} 1 & -2 & 3 \\ 4 & 5 & -6 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 0 & 2 \\ -7 & 1 & 8 \end{pmatrix}$ Marks 3

b) Given $A = \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 0 \\ -6 & 7 \end{pmatrix}$ Marks 2

Find $(AB)^T$

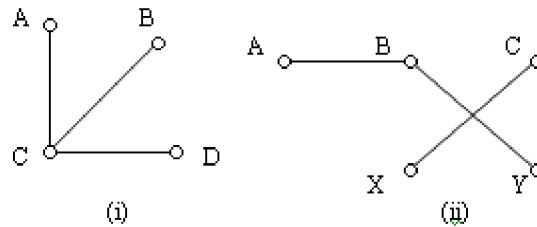
c) Let $A = \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}$ Marks 3

Find A^3

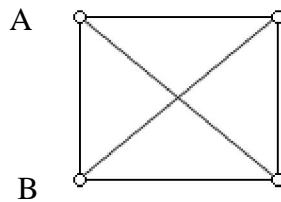
d) Show that $A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix}$ and $B = \begin{pmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{pmatrix}$ are inverses. Marks 2

e) Find the inverse of $\begin{pmatrix} 3 & 5 \\ 2 & 3 \end{pmatrix}$ Marks 4

f) Determine whether or not each of the graphs in figure is connected. Marks 2



g) Draw a diagram of the complete graph K_4 Marks 2



h) Draw the complete bipartite graph $K_{2,3}$ Marks 2

Question four (Marks 20)

a) Draw the graph G whose adjacency matrix $A = (a_{ij})$ follows

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$
Marks 2

b) Find the number m of ways that 9 students can be divided evenly into three teams.

- c) Find the truth table of $\sim p \wedge q$ Marks 2
Marks 3
- d) Verify that the following proposition $p \vee \sim (p \wedge q)$ is a tautology. Marks 4
- e) Show that the propositions $\sim (p \wedge q)$ and $\sim p \vee \sim q$ are logically equivalent. Marks 4
- f) Construct the truth table of $\sim p \rightarrow (q \rightarrow p)$ Marks 4
- g) Compute $\frac{13!}{11!}$ Mark 1

Question five (Marks 20)

- a) Draw a logic circuit diagram corresponding to the Boolean expression $Y = \overline{A + BC} + B$ Marks 4
- b) Use karnaugh maps to find the prime implicants and minimal form for each of the following complete sum-of-products Boolean expression.
 $E = xy + \overline{x}y + \overline{xy}$ Marks 4
- c) Design a minimal AND-OR circuit which yield the following truth table
 $T = [A=00001111, B=00110011, C=01010101; L=10101001]$ Marks 8
- d) Find $\|W\|$ if $W = (-3, 1, -2, 4, -5)$ Mark 1
- e) Let $u = (5, 4, 1)$, $v = (3, -4, 1)$ and $w = (1, -2, 3)$ which of the vectors, if any, are orthogonal? Marks 3