KABARAK UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **PHYS 323**

COURSE TITLE: **ELECTROMAGNETIC THEORY**

STREAM: SESSION VI

DAY: **WEDNESDAY**

TIME: **2.00 – 4.00 P.M**

DATE: **08/04/2009**

INSTRUCTIONS

Answer questions **ONE** and any other **TWO**. Question ONE carries 30 marks, and the others 20 marks each.

$\mu_o = 4\pi x 10^{-7}\Omega$, $\varepsilon_o = 8.85x10^{-12}$, Rayleigh scattering cross-section is $\sigma_{SR} = \frac{32\pi}{3} R^2 \left(\frac{\omega}{\omega_o}\right)^4$,

Resistance of a slab at a high frequency is $R = \frac{L}{\sigma S}$, where symbols have their usual meanings.

$\nabla \times (\nabla \times E) = \nabla(\nabla . E) - \nabla^2 E$, $\nabla.(E \times H) = -E.\nabla \times H + H.\nabla \times E$

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Question 1 (30 marks)

(a) (i) Mention the three sources of magnetic moments in an atom. (1½ marks)

(ii) Sketch spins of

I. An antiferromagnetic material

II. A ferrimaget (1, 1 mark)

(iii) Distinguish between the three types of current densities in a material. (1½ marks)

(b) Consider a magnetic dipole **m** between two rectangular-shaped regions of different magnetizations as shown in figure 1.

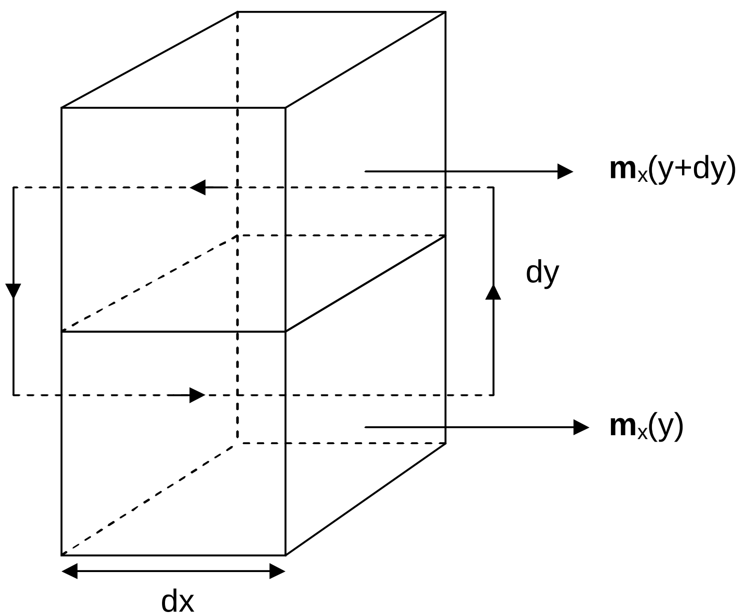

Figure 1: magnetic dipole between two regions of unequal magnetization.

Show that for this scenario, the current density in the z-direction is $J_Z^{'} = \nabla \times M$.

(3 marks)

(c) Consider a situation whereby you have a magnetized material as well as steady currents. Show that the magnetic field intensity **H** is, $H = \dfrac{B}{\mu_o} - M$, and hence $\int_s (\nabla \times H).da = \int_s J_f .da$, where the symbols have their usual meanings. (2 marks)

(d) Write down the Maxwell's equations in integral form. (4 marks)

(e) (i) Starting with the Maxwell's equation which relates the displacement vector **D** with the charge density ρ, derive Poisson's equation for the potential V. Hence derive the Laplace equation for regions with no charge density. (2, 1 marks)

(ii) Mention two types of waveguides. (2 marks)

(f) A conductor has a circular cross-section of radius 2.5mm and is constructed from steel for which $\sigma=5.1\text{x}10^6$mhos/m and $\mu_r=200$. If the conductor is 300m long and carries a total current I (t) =1.5cos3x10^4tA, determine

(i) the effective resistance
(ii) the resistance for a dc current (2, 2 marks)
(iii) Define an electret. (1 mark)

(h) (i) A 4-GHz uniform plane wave is normally incident from region 1, z<0, $\varepsilon_{r1}=5$, $\mu_{r1}=1$, $\sigma_1=0$, toward region 2, z>0, $\varepsilon_{r2}=2$, $\mu_{r2}=10$, $\sigma_2=0$. Find (a) S in regions 1 (b) the transmission coefficient. Assume $\eta = \sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\varepsilon}}$. (3, 2 marks)

(ii) Explain why only a thin layer is required as a waveguide for a good conductor. (1 mark)

<u>Question 2 (20 marks)</u>

(a) (i) Sketch the electric lines of force (field) between two charges whose magnitudes are $2Q^+$ and Q^+. (1 mark)

(b) (i) Consider the case of a time-constant magnetic flux **B**, and a moving closed path as shown in figure 2.

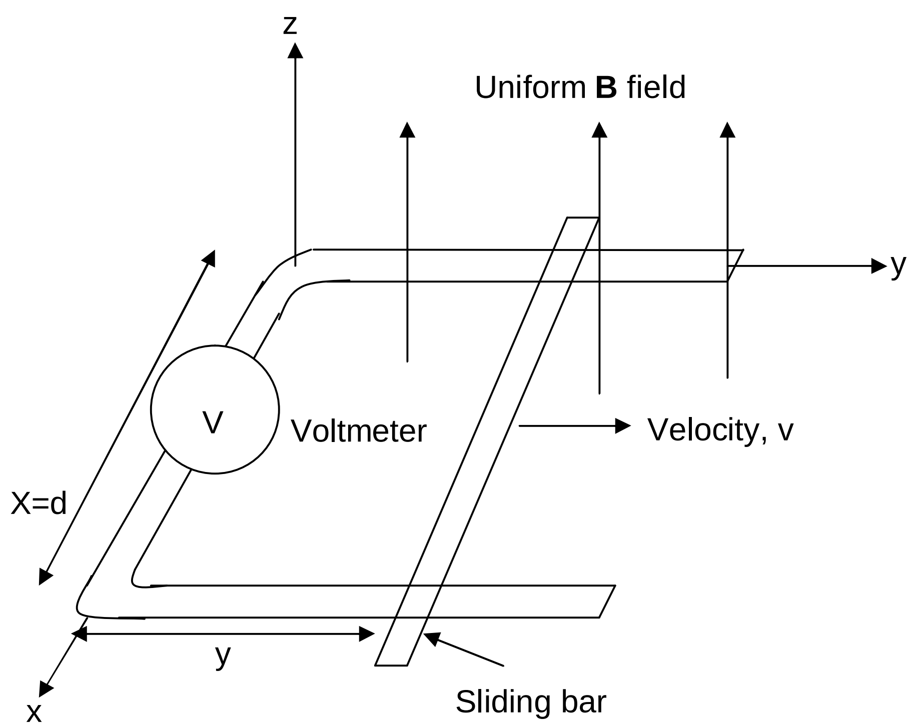

Figure 2; A moving closed path in a fixed **B** field.

Show that the second Maxwell's equation is $\nabla \times \mathsf{H} = \mathsf{J} + \frac{\partial \mathsf{D}}{\partial t}$, where the symbols have their usual meanings. (5½ marks)

(ii) Write down the other three Maxwell's equation in differential form. (2 marks)

(iii) Write the equivalent of the Maxwell's in electric fields. (2 marks)

(c) (i) Show that $\eta = \sqrt{\frac{\mu}{\varepsilon}} \frac{1}{\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}}} = \sqrt{\frac{\mu}{\varepsilon}}\left[1 + j\frac{\sigma}{2\omega\varepsilon}.\right]$ if the loss tangent is very small. The symbols have their usual meanings. (2½ marks)

(ii) Suppose a 300MHz wave is traveling through fresh water which is considered as a lossless medium. If μ_r=1 and ε_r=78, determine

I. The velocity of propagation v of the wave (2 marks)
II. The wavelength of the wave in free space. (2 marks)
III. The phase constant, β. (2 mark)
IV. The intrinsic impedance, η. (2½ marks)

Question 3 (20 marks)

(a) (i) Derive the four Maxwell's equations in terms of phasor-vectors given that $\mathsf{E}_s = \mathsf{E}_{xs} e^{j\omega t}$ and $\mathsf{H}_s = \mathsf{H}_{xs} e^{j\omega t}$. (4 marks)

(ii) Hence derive the vector Helmholtz equation. (3 marks)

(iii) Show that $E_{xs} = Ae^{\pm j(kz+\alpha)}$ is a solution of the vector Helmholtz equation for $k = \omega\sqrt{\mu_o \varepsilon_o}$ at any α. (3 marks)

(b) Consider the x component of the electric field component written as $\mathsf{E}_{xs} = \mathsf{A}e^{-j\omega\sqrt{\mu_o\varepsilon_o}z}$.

(i) Show that the real part of the electric field component of a plane wave in free space traveling in the x-direction is $\mathsf{E}_x = \mathsf{A}\cos\omega(t - \sqrt{\mu_o \varepsilon_o}z)$, where the symbols have their usual meanings. (3 marks)

(ii) Assuming the E_x directed upwards at the surface of plane earth, determine the speed of light in free space. (2 marks)

(iii) Suppose that a location B is 1000km to the east of another location A, find the field strength at point B in relation to that at point A. (2 marks)

(c) Using the relations $\frac{\partial E_{xs}}{\partial z} = -j\omega\mu_o H_{ys}$ and $E_{xs} = E_{xo} e^{-j\omega\sqrt{\mu_o\varepsilon_o}z}$, show that $\frac{E_x}{H_y} = \text{constant}$. (3 marks)

<u>Question 4 (20 marks)</u>

(a) Sketch the B and H field lines inside a bar magnet. (2 marks)

(b) Starting with the plane wave equation that, $\nabla^2 E = -\omega^2\mu\varepsilon E$, show that for a plane wave propagating in a perfect dielectric

(i) the phase constant β is, $\beta = \omega\sqrt{\mu\varepsilon}$

(ii) Derive the expression for the wavelength of a plane wave propagating in the material in terms of its corresponding wavelength in free space λ_o. (4, 3 marks)

(c) Starting with the Maxwell's equation $\nabla \times H_s = J + \frac{\partial D}{\partial t}$, show that

$$\oint_s (E \times H).dS = \int_{vol} J.E dV + \frac{\partial}{\partial t}\int_{vol}\left(\frac{\varepsilon E^2}{2} + \frac{\mu H^2}{2}\right)dV$$ (3½ marks)

(d) Show that the time average power density in one cycle according to the Pointing vector is, $\frac{1}{2}\frac{E_{xo}^2}{\eta} w/m^2$. Assume $E_x = E_{xo}\cos(\omega t - \beta z)$ (3 marks)

(e) (i) Given that the general expression for the propagation constant γ is

$\gamma = j\omega\sqrt{\mu\varepsilon}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}}$, show that for a good conductor $\alpha = \beta = \sqrt{\pi f\mu\sigma}$. (3½ marks)

(ii) Hence, find an expression for the skin depth δ. (1mark)

<u>Question 5 (20 marks)</u>

(a) Distinguish between TE, TM and TEM waves. (1½ marks)

(b) Show that for a good conductor, (i) $\frac{\delta}{a_0} <<< 1$ (2 marks)

(ii) Sketch a rectangular showing the directions of the wave vector, **k**, $\mathbf{E_m}$ and $\nabla\mathbf{H_{mz}}$.
(1½ marks)

(iii) Sketch diagrams illustrating of linearly and elliptically polarized waves. (2 marks)

(c) Consider a incident uniform plane wave being reflected and transmitted at a boundary between two regions as shown in figure 3

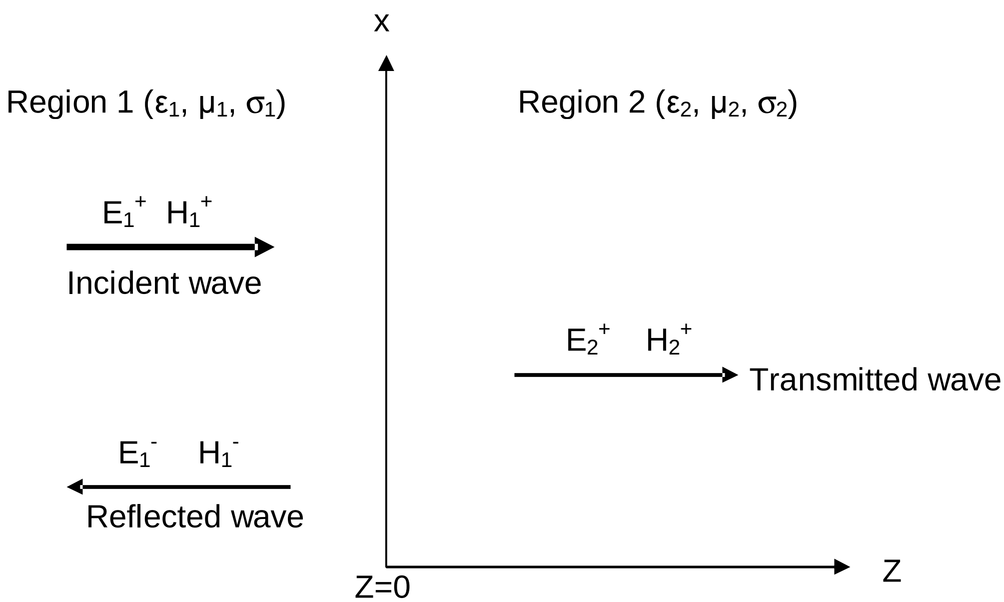

Figure 3: A uniform plane wave being reflected and transmitted at an interface of two regions.

(i) Given that the electric and magnetic field components of the incident and reflected waves in regions 1 are $E_{xs1}^{+} = E_{x10}^{+} e^{-\gamma_1 z}$, $H_{ys1}^{+} = \frac{E_{xs1}^{+}}{\eta_1} = \frac{E_{x10}^{+}}{\eta_1} e^{-\gamma_1 z}$ and $E_{xs1}^{-} = E_{x10}^{-} e^{\gamma_1 z}$, $H_{ys1}^{-} = -\frac{E_{x10}^{-}}{\eta_1} e^{\gamma_1 z}$ respectively, while the transmitted wave in region 2 has components, $E_{xs2}^{+} = E_{x20}^{+} e^{-\gamma_2 z}$, $H_{ys2}^{+} = \frac{E_{xs2}^{+}}{\eta_2} = \frac{E_{x20}^{+}}{\eta_2} e^{-\gamma_2 z}$, show that $E_{x10}^{-} = E_{x10}^{+} \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$,

(3½ marks)

(ii) Find an expression for the reflection coefficient, Γ. (1 mark)

(iii) Find an expression for the transmission coefficient. (1 mark)

(iv) Given that η_1=300Ω, η_2=100Ω and $E_{x10}^{+} = 100 V/m$, find

I. Γ

II. E_{x10}^{-}

III. H_{y10}^{+}

IV. H_{y10}^{-}

V. The incident average power density $P_1^{+}(av.)$

VI. The reflected average power density $P_1^{-}(av.)$

(1, 1, 1, 1, 1, 1 marks)

(d) Show that the Rayleigh scattering cross-section of an electromagnetic wave is proportional to $(\lambda^4)^{-1}$, where λ is the wavelength of the wave. (1½ marks)