KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2010/2011 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN TELECOMMUNICATIONS

COURSE CODE: MATH 216

COURSE TITLE: ENGINEERING MATHEMATICS

STREAM: Y2S1

DAY: THURSDAY

TIME: 9.00 – 12.00 P.M.

DATE: 16/12/2010

INSTRUCTIONS:

- **Answer Question ONE and any other THREE Questions.**

PLEASE TURN OVER

QUESTION ONE (20 MARKS)

a) A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$ and $z = 2\sin 3t$, where t is time.

   a). Determine its velocity and acceleration at any time. (3 marks)

   b). Find the magnitude of the velocity and acceleration at $t = 0$. (2 marks)

b) Use Runge-Kutta method to obtain an approximation of $y(1.2)$ for the solution of the function $y' = 2xy$ with the initial condition $y(1) = 1$ using a step size of $h = 0.1$.

(6 marks)

c) Given that $\tilde{A} = (2x^2 y - x^4)\hat{i} + (e^{xy} - y\sin x)\hat{j} + (x^2 \cos y)\hat{k}$. Find $\frac{\partial^2 \tilde{A}}{\partial x \partial y} + 2\frac{\partial^2 \tilde{A}}{\partial x^2} - \frac{\partial^2 \tilde{A}}{\partial y^2}$.

(4 marks)

d) Find the solution of the following initial value problem of the second order equation

$$y'' - 5y' + 6y = 0, \quad y(0) = 1, \quad y'(0) = -1$$

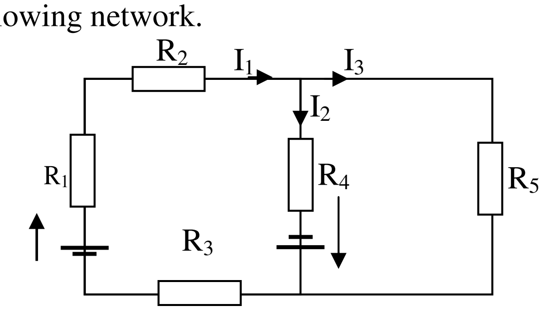
(5 marks)

QUESTION TWO (10 MARKS)

a) Use Euler method to obtain the approximate value of $y(0.2)$ for the solution of $y' = (x + y - 1)^2$ with the condition $y(0) = 2$ and for a step function of $h = 0.1$

(3 marks)

b) Consider the following network.



i). Use Kirchoff's law to determine the equations for the unknown currents I_1, I_2 and I_3.

(3 marks)

ii). Using Cramer's rule, express I_1, I_2 and I_3 in terms of R_1, R_2, R_3, R_4 and R_5. (4 marks)

QUESTION THREE (10 MARKS)

a) Solve the initial value problem $(1 + y^2)dx + (1 + y^2)dy = 0$ using the initial conditions $y(0) = -1$. (3 marks)

b) Find the integrating factor of the following differential equation and solve it.

$$2x^2 y dx + (x^3 + 2xy)\, dy = 0$$

(3 marks)

c) Consider the following differential equation that models the harmonic oscillator $m\ddot{y}+c\dot{y}+ky=0$, where c is the coefficient of damping, k is spring constant and m is the mass of the object attached to the spring. Show that the general solution of the system in case of under-damping is given by $y(t)=e^{-\alpha t}(A\cos\omega t+B\sin\omega t)$ where A and B are arbitrary constants of integration and $\alpha=\dfrac{c}{2m}$ and $\omega=\frac{1}{2m}\sqrt{4mk-c^2}$ (4 marks)

QUESTION FOUR (10 MARKS)

a) Find the solution of the wave equation by separation of variables $\alpha\dfrac{\partial^2 u}{\partial x^2}=\dfrac{\partial^2 u}{\partial t^2}$, $0<x<L$, $t>0$ with the conditions

$$u(0,t)=0, \qquad u(L,t)=0, \qquad t\geq 0 \text{ and}$$

$$u(x,0)=f(x), \qquad \frac{\partial u}{\partial t}\Big|_{t=0}=g(x), \qquad 0<x<L,$$ (6 marks)

b) Expand $f(x)=x^2$ using Half-Range Fourier cosine Series.

(4 marks)

QUESTION FIVE (10 MARKS)

a) A particle moves along the curve $\tilde{r}=(t^3-4t)\hat{i}+(t^2+4t)\hat{j}+(8t^2-3t^3)\hat{k}$ where t is time. Find the magnitude of the tangential and normal components of its acceleration when $t=2$. (5 marks)

c) Evaluate $\iint_S \hat{F}.\hat{n}dS$ where $\hat{F}=z\hat{i}+x\hat{j}+y^2z\hat{k}$ and S is the surface of the cylinder $x^2+y^2=4$ in the first octant between $z=0$ and $z=4$. (5 marks)