KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2010/2011 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF TELECOMMUNICATION

COURSE CODE: **MATH 216**

COURSE TITLE: **ENGINEERING MATHEMATICS**

STREAM: **Y2S1**

DAY: **THURSDAY**

TIME: **9.00 – 12.00 P.M.**

DATE: **24/03/2011**

INSTRUCTIONS:

1. Question ONE is compulsory.
2. Attempt question ONE and any other THREE

PLEASE TURN OVER

QUESTION ONE-COMPULSORY (40 marks)

a) Find the unit tangent vector to any point on the curve $x = t^2 + 1$, $y = 4t - 3$ and $z = 2t^2 - 6t$ hence find the unit tangent at the point where t = 2 **(3 marks)**

b) Determine the constant a so that the vector $\vec{V} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}$ is a solenoid

(2 Marks)

c) Evaluate the iterated volume integral $\int_0^1 \int_0^{2x} \int_{x-y}^{x+y} 30xyz\,dz\,dy\,dx$ **(4 marks)**

d) Verify green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2 dy$ where C is the closed curve bounded by $y = x$ and $y = x^2$ **(2 marks)**

e) Determine the particular solution of $(y^2 - 1)\frac{dy}{dx} = 3y$ given that $y = 1$ when $x = 2\frac{1}{6}$

(6 marks)

f) Determine the solution of the following non-homogeneous equation by the method of variation of parameters $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{1}{1+e^{-x}}$ **(6 marks)**

g) Determine $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ when $z = \frac{1}{\sqrt{(x^2 + y^2)}}$ **(2 marks)**

h) Verify that $u(\theta, t) = \theta^2 + \theta t$ is a solution of $\frac{\partial u}{\partial \theta} - 2\frac{\partial u}{\partial t} = t$ **(4 marks)**

i) Find two linear independent solutions to the system;

$x^1 = Ax$ for A = $\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$ **(6 marks)**

j) Determine the Half-Range Fourier cosine Series to represent $f(x) = 3x$ in the range $0 \leq x \leq \pi$ **(5 marks)**

QUESTION TWO: 20 MARKS

a) A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$ and $z = 2\sin 3t$ where t is the time. Determine the velocity and acceleration at any given time and hence find the magnitude of the velocity and acceleration at t = 0 **(3 marks)**

b) A space curve is given by the equations $x = t$, $y = t^2$, $z = \frac{2}{3}t^3$ find **(6 marks)**

i) the curvature κ

ii) The torsion τ .

c) If $A = (3x^2 + +6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$ evaluate the line integral $\int_C \vec{A} \bullet d\underset{\sim}{r}$ from $(0,0,0)$ to $(1,1,1)$ along the following path, $x = t$, $y = t^2$ $z = t^3$ **(3 Marks)**

d) Given that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$

i) Show that $\vec{F}$ is a conservative force field. **(1mark)**

ii) Find the scalar potential **(2marks)**

iii) Find the work done in moving an object in this field from $(1,-2,1)$ to $(3,1,4)$

(1 mark)

e) Use divergence theorem to evaluate $\oiint_S \breve{r} \bullet \hat{n}ds$ where $\breve{r} = 2x\hat{i} + zy\hat{j} + x\hat{k}$ and S is the surface of tetrahedron bounded by the plane $x = 0$ $z = 0$ $x + y + z = 1$ **(4 marks)**

QUESTION THREE: 20 MARKS

a) The variation of resistance R ohms, of an aluminium conductor with temperature $\theta\,^0C$ is given by $\frac{dR}{d\theta} = \alpha R$ where α is the temperature coefficient of resistance of aluminium.

i) Given that $R = R_0$ when $\theta = 0^0C$ find the particular solution **(4 marks)**

ii) Determine the resistance of an aluminium conductor at 50^0C given that its resistance at 0^0C is $24\cdot 0\Omega$ take $\alpha = 38\times 10^{-4}/^0C$ **(2 marks)**

b) The equation $\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$ represents a current i flowing in an electrical circuit containing resistance R inductance L and capacitance C connected in series. Given that $R = 200$ ohms, $L = 0\cdot 20$ henry and $C = 20\times 10^{-6}$ farads solve the equation for i given the boundary conditions that when $t = 0$, $i = 0$ and $\frac{di}{dt} = 100$. **(4 marks)**

c) An equation used in thermodynamics is the benedict-Webb-Rubine equation of state for the expansion of gas. The equation is

$$p = \frac{RT}{V} + \left(B_0RT - A_0 - \frac{C_0}{T^2}\right)\frac{1}{V^2} + (bRT - a)\frac{1}{V^3} + \frac{A\alpha}{V^6} + \frac{C\left(1 + \frac{\gamma}{V^2}\right)}{T^2}\left(\frac{1}{V^3}\right)e^{-\frac{\gamma}{V^2}}$$

Show that $\frac{\partial^2 p}{\partial T^2} = \frac{6}{V^2T^4}\left\{\frac{C}{V}\left(1+\frac{\gamma}{V^2}\right)e^{-\frac{\gamma}{V^2}} - C_0\right\}$ **(5 marks)**

d) A square plate is bounded by the lines $x = 0$ $y = 0$ $x = 1$ and $y = 1$. Apply the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ to determine the potential distribution $u(x, y)$ over the plate subject to the following boundary conditions. **(5 marks)**

$u = 0$ when $x = 0$ $\qquad 0 \le y \le 1$

$u = 0$ when $x = 1$ $\qquad 0 \le y \le 1$

$u = 0$ when $y = 0$ $\qquad 0 \le x \le 1$

$u = 4$ when $y = 1$ $\qquad 0 \le x \le 1$

QUESTION FOUR: 20 MARKS

a) Solve the system **(7 marks)**

$$2x - 3y + z = 1$$
$$x - 6y + 3z = -4$$
$$3x + 3y - 2z = 7$$

b) The following information system of equations governs the vertical vibrations of two coupled springs with masses m_1 and m_2 suspended on them.

$$m_1 y' = -k_1 y_1 + k_2(y_2 + y_1)$$
$$m_2 y' = -k_2(y_2 + y_1)$$

Where k_1 and k_2 are spring constants and y_1, y_2 are the displacements due to extension by the spring and y' denote differentiation with respect to time.

i) Write the system in the form $y' = Ay$ where A is the coefficient matrix **(3 marks)**

ii) Let $m_1 = m_2 = 1$, $k_1 = 3$, and $k_2 = 2$, find the eigenvalues and the corresponding eigenvectors of the system in (i) above. **(6 marks)**

iii) Obtain the solution of the system in (ii). above using the initial conditions $y_1(0) = 1,\ y_2(0) = 2,\ \hat{y}_1(0) = -2\sqrt{6},\ \hat{y}_2(0) = \sqrt{6}$. **(4 marks)**

<u>QUESTION FIVE: 20 MARKS</u>

a) Deduce the Fourier series for the function $f(\theta) = \theta^2$ in the range 0 to 2π **(6 marks)**

b) The voltage from a square wave generator is of the form:

$$v(t) = \begin{cases} 0, \ldots\ldots\ldots\ldots -4 < t < 0 \\ 10, \ldots\ldots\ldots\ldots\ldots 0 < t < 4 \end{cases}$$

Find the Fourier series for this periodic function. **(5 marks)**

c) Given the differential equation $\frac{dy}{dx} = y - x$

i) Obtain a numerical solution using Euler's method with the initial conditions $x = 0$ $y = 2$ for the range $x = 0(0 \cdot 1)0 \cdot 5$ **(4 marks)**

ii) Draw the graph of the solution **(2 marks)**

iii) Using integrating factor method find the solution of the differential equation **(2 marks)**

iv) Determine the percentage error at $x = 0 \cdot 3$ **(1 mark)**