KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/2010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

COURSE CODE: **MATH 112**

COURSE TITLE: **GEOMETRY AND ELEMENTARY APPLIED MATHEMATICS**

STREAM: **Y1S1**

DAY: **THURSDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **25/03/2010**

INSTRUCTIONS:

1. Question **ONE** is compulsory.
2. Attempt question ONE **and any other TWO**

PLEASE TURN OVER

Question One [30 Marks]

a) Given that **a** = (15, – 6, 24), **b** = (5, – 2, 8) and **c** = (– 15/2, 3, – 12) show that **a** and **b** have the same direction and **a** and **c** have the opposite direction. [3 marks]

b) Find the equation of the line that is perpendicular to the line 5x – y + 8 = 0 and passes through the point of intersection of the lines

2x + 7y – 3 = 0 and 3x – 2y + 8 = 0. [5 marks]

c) Solve the following simultaneous equation by use matrix algebra.

$$2x - y = 4$$
$$4x + y = 5$$

[3 marks]

d) Determine the equation of a circle whose center is (– 1, 1) and it is tangent to the line x + 2y = 4 [4 marks]

e) Find the equation of an ellipse with eccentricity $^2/_3$ given that the line x = 9 is one of the directrix and the corresponding focus is at (4, 0). [4 marks]

f) Reduce the equation $5x^2 - 4y^2 + 20x + 8y = 4$ to standard form. Identify the conic and give the coordinates if its foci and vertices. [5 marks]

g) Simplify completely the expression $\left(5 - \sqrt{-9}\right)\left(-1 + \sqrt{-4}\right)$ [3 marks]

h) Find an equation in x and y that has the same graph as the polar equation[3 marks]

$$r = \frac{15}{4 - 4\cos\theta}$$

Question Two [20 Marks]

a) Find the equations of the lines through the point (4, 2) and at a perpendicular distance 2 units from the origin. [5 marks]

b) Find the vertices, foci, axis of symmetry for the graph represented by the polar equation $r = \dfrac{10}{3 + 2\cos\theta}$ [10 marks]

c) A parabola intersects a rectangle of area A at two opposite vertices. Show that, if one side of the rectangle falls on the axis of the parabola, then the parabola subdivides the rectangle into two pieces, one of area $^1/_3$A, the other of area $^2/_3$A. [5 marks]

Question Three [20 Marks]

a) Consider the lines 7x + 2y = 7 and 2x – 3y = 27. Find

i) The angle between the lines

ii) The distance from their point of intersection to the line x = 3y + 5 [6 marks]

b) If the line x = 2y meets the circle $x^2 + y^2 - 8x + 6y - 15 = 0$ at the points P and Q find

i) The co-ordinates of P and Q [3 marks]

ii) The equation of the circle passing through P, Q and the point (1, 1) [4 marks]

iii) The equation of the tangent of the circle in (ii) above at the point (1, 1) [3 marks]

c) A cruise ship is traveling a course that is 100 miles from and parallel to a straight shoreline. The ship sends out distress signal, which is received by two coast guards stations A and B, located 200 miles apart. By measuring the difference in signal reception times, officials determine that the ship is 160 miles to B than A. Find the location of the ship. [4 marks]

Question Four [20 Marks]

a) Find the six sixth roots of -1 [7 marks]

b) Find the values of a and b such that $(a+ib)^2 = i$. Hence or otherwise solve the equation $z^2 + 2z + 1 - i = 0$ giving your answer in the form $p + iq$ where p and q are real numbers. [7 marks]

c) Show that the scalar triple product $\underline{a}.(\underline{b} \times \underline{c})$ is given by $\det\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ [6 marks]

Question Five [20 Marks]

a) Find the area of the triangle determined by $P(4,-3,1)$, $Q(6,-4,7)$ and $R(1,2,2)$ [4 marks]

b) Two products , maize and beans, currently share the market with the shares of 60% and 40% respectively. Each week, some proportion switching takes place. Of those who bought maize the previous week, 70% buy it again and 30% switch to beans. Of those who bought beans the previous week, 80% buy it again and 20% switch to maize. Find the proportion of the market the products will finally hold? [5 Marks]

c) Product x has fixed cost of £ 60 and variable cost of £ 6 per product; product y has fixed costs £ 35 and variable costs of £ 8 per product.

If $C = \begin{pmatrix} 60 & 6 \\ 35 & 8 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 \\ q \end{pmatrix}$ where C is the cost coefficient matrix.

i) Evaluate and explain the significance of the matrix CQ [3 marks]

ii) Given that he products x and y sell at £ 9 and £ 10 respectively write down the revenue coefficient matrix R [2 marks]

iii) Evaluate RQ – CQ and explain its significance. [3 marks]

iv) I P = RQ – CQ $A = \begin{pmatrix} 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \end{pmatrix}$ solve AP = BP and interpret the value of q obtained. [3 Marks]