KABARAK   **UNIVERSITY**

UNIVERSITY EXAMINATIONS
2009/2010 ACADEMIC YEAR
FOR THE DEGREE OF BACHELOR OF SCIENCE
IN ECONOMICS AND MATHEMATICS

COURSE CODE: MATH 112

COURSE TITLE: GEOMETRY AND ELEMENTARY
                                    APPLIED MATHEMATICS

STREAM: **Y1S1**

DAY: **FRIDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **13/08/2010**

INSTRUCTIONS:

1.Question ONE is compulsory.

2. Attempt question ONE and any other TWO Questions

PLEASE TURNOVER

Question One   [30 Marks]

a)   Define the following terms

    i)   An ellipse

    ii)   A matrix

    iii) A Complex number.   [3 marks]

 Find the equation of the line that is perpendicular to the line $5x - y + 8 = 0$ and passes through the point of intersection of the lines

        $2x + 7y - 3 = 0$ and $3x - 2y + 8 = 0$.   [5 marks]

b)   Solve the following simultaneous equation by use matrix algebra.

$$2x - y = 4$$
$$4x + y = 5$$
                                            [3 marks]

c)   Determine the equation of a circle whose center is $(-1, 1)$ and it is tangent to the line $x + 2y = 4$ [4 marks]

d)   Find the equation of an ellipse with eccentricity $^2/_3$ given that the line $x = 9$ is one of the directrix and the corresponding focus is at $(4, 0)$.   [4 marks]

e)   Reduce the equation $5x^2 - 4y^2 + 20x + 8y = 4$ to standard form. Identify the conic and give the coordinates if its foci and vertices.   [5 marks]

f)   Simplify completely the expression $\left(5 - \sqrt{-9}\right)\left(-1 + \sqrt{-4}\right)$ [3 marks]

g)   Eliminate the parameter and deduce the resulting conic.

    $x = a\cos t + h$, $y = b\sin t + k$ [3 marks]

Question Two   [20 Marks]

a)   Find the equations of the lines through the point $(4, 2)$ and at a perpendicular distance 2 units from the origin.   [6 marks]

b)   A point moves in a plane its position $P(x, y)$ at time t is given by $x = 5\cos t$ and $y = 5\sin t$; $t \in \Re$. Describe the motion of the point.   [5 marks]

c)   Find the center, the foci, the length of major and minor axis of the ellipse.

    $16x^2 + 25y^2 - 64x - 150y - 111 = 0$ [6 marks]

d)   A sound-receiving dish used at outdoor sporting events is constructed in the shape of a paraboloid with its focus 12.5cm from the vertex. Determine the width of the dish if the depth is to be 5cm.   [3 marks]

Question Three [20 Marks]

a) Consider the lines $7x + 2y = 7$ and $2x - 3y = 27$. Find

     i)   The angle between the lines

     ii)   The distance from their point of intersection to the line $x = 3y + 5$

                                                      [6 marks]

b) If the line $x = 2y$ meets the circle $x^2 + y^2 - 8x + 6y - 15 = 0$ at the points P and Q find

     i)   The co-ordinates of P and Q   [4 marks]

     ii)   The equation of the circle passing through P, Q and the point (1, 1)

                                                        [6 marks]

c) A cruise ship is traveling a course that is 100 miles from and parallel to a straight shoreline. The ship sends out distress signal, which is received by two coast guards stations A and B, located 200 miles apart. By measuring the difference in signal reception times, officials determine that the ship is 160 miles to B than A. Find the location of the ship.   [4 marks]

Question Four [20 Marks]

a) Find the six sixth roots of -1 [7 marks]

b) Find the values of a and b such that $(a + ib)^2 = i$. Hence or otherwise solve the equation $z^2 + 2z + 1 - i = 0$ giving your answer in the form $p + iq$ where p and q are real numbers.   [7 marks]

c) The equation $x^4 - 4x^3 + 3x^2 + 2x - 6 = 0$ has a root $1 - i$. Find the other roots

                                                      [6 marks]

Question Five [20 Marks]

a) Find the area of the triangle determined by $P(4,-3,1)$, $Q(6,-4,7)$ and $R(1,2,2)$

                                                   [4 marks]

b) If α and β are the roots of $z^2 - 10z + 29 = 0$ find

     i) $\alpha + \beta$

     ii) $\alpha\beta$ [5 Marks]

c) A group operates a chain of shops in each of which are employed cashiers, attendants and cleaners as shown

Types of shop

	Large	**Medium**	**Small**
Cashiers	4	2	1
Attendants	12	6	3
Cleaners	6	4	2

The number of shops is

	Eastern Kenya	**Western Kenya**
Large	3	7
Medium	5	8
Small	12	4

How many of the various types of staff are employed in Eastern Kenya and Western Kenya? [10 mks]