KABARAK UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **PHYS 311**

COURSE TITLE: **MATHEMATICAL PHYSICS**

STREAM: **SESSION V**

DAY: **SATURDAY**

TIME: **9.00-11.00 A.M.**

DATE: **29/11/2008**

INSTRUCTIONS:

This examination paper consists of five questions. Question ONE carries 40 marks while each of the other questions carries 15 marks. Answer question ONE and any other TWO questions.

PLEASE TURN OVER

Question 1 (40marks)

a) Distinguish between a vector and a scalar quantity giving an example of each. (3mks)

b) An operator $\hat{A}$ is represented by the matrix

$$\hat{A} = \begin{pmatrix} 0 & -i & -3i \\ i & 5 & 0 \\ 3i & 0 & 2 \end{pmatrix}$$

where i denotes the complex notation. Determine whether or not the operator is Hermitian. (3mks)

c) (i) Define the term 'boundary conditions.' (1mk)
(ii) Hence briefly describe the three forms of boundary conditions. (3mks)

d) State two properties of the Laplace transform. Hence evaluate the Laplace transform of $t^2 e^{-4t} + 2t^2$. (4mks)

e) The primitive translation vectors of the hexagonal close-packed crystal structure in the direct lattice are given by $\bar{a} = a\hat{x}$, $\bar{b} = \left(\frac{a}{2}\right)\hat{x} + \left(\frac{\sqrt{3}}{2}\right)\hat{y}$ and $\bar{c} = c\hat{z}$ where $\hat{x}$, $\hat{y}$ and $\hat{z}$ are the unit vectors in the three axial directions respectively. Determine the reciprocal lattice vector

$$\bar{a}^* = 2\pi \frac{\bar{b} \times \bar{c}}{\bar{a} . (\bar{b} \times \bar{c})}$$

f) The equation of motion for a linear harmonic oscillator is given by; $\frac{d^2x}{dt^2} + \omega^2 x = 0$ where x is the displacement from some reference point and ω is the angular frequency. Formulate the characteristic equation of the differential equation and find the general expression for the displacement x of the oscillator. (4mks)

g) A force field $\bar{F} = -x^2\hat{x} + y^3\hat{y}$ acts on a particle. Determine the work done against the force in moving the particle from (0,0) to (5,5) along the straight-line path (0,0) to (5,0) to (5,5). (4mks)

h) A boat is sailing at a velocity of 40km/hr north. Water current is flowing at 30km/hr north east. Determine the resultant velocity of the boat. (5mks)

i) Briefly explain, with an example, the physical meaning of divergence. (2mks)

j) (I) Define the term 'the Langrangian' of a non-relativistic system. (1mk)
(II) State Hamilton's Principle of Classical Mechanics as applied to calculus of variation. (2mks)

k) Two particles start from the same point and move in two different directions represented by the vectors $\overline{a} = 3\hat{x} - 2\hat{y} + 2\hat{z}$ and $\overline{b} = 2\hat{x} + 3\hat{y} + 2\hat{z}$ respectively. Find the angle between their directions of motion. (3mks)

<u>Question 2 (15 marks)</u>

a) Define each of the following terms:
(i) An irrotational vector
(ii) A solenoidal vector field (2mks)

b) The angular momentum $\overline{L}$ of a body is defined as $\overline{L} = \overline{r} \times \overline{p}$ where $\overline{r}$ is the radius vector and $\overline{p}$ is the linear momentum. Given that $\overline{p} = -i\overline{\nabla}$ where i is the complex notation and $\overline{\nabla}$ is the 'del' operator, determine the Cartesian components of $\overline{L}$.
(6mks)

b) A magnetic vector potential $\overline{A}$ is given by $\overline{A} = -\frac{\mu_0 I}{4\Pi} \ln\left(x^2 + y^2\right)\hat{z}$ where I is the current flowing through a long straight wire.
(i) Determine the magnetic induction $\overline{B}$ due to the current I in the wire. (4mks)
(ii) Hence determine whether or not $\overline{B}$ is solenoidal. (3mks)

Question 3 (15 marks)

a) (i) Define the term a ‘complex variable’ giving one example. (2mks)

(ii) Given two complex numbers $Z_1 = 2 + 3i$ and $Z_2 = 5 + 2i$, show that;

(I) $|Z_1 + Z_2| \angle |Z_1| + |Z_2|$ (3mks)

(II) $\arg(Z_1.Z_2) = \arg Z_1 + \arg Z_2$ (3mks)

c) (i) State Cauchy-Riemann’s conditions for a complex function of a complex variable. (2mks)

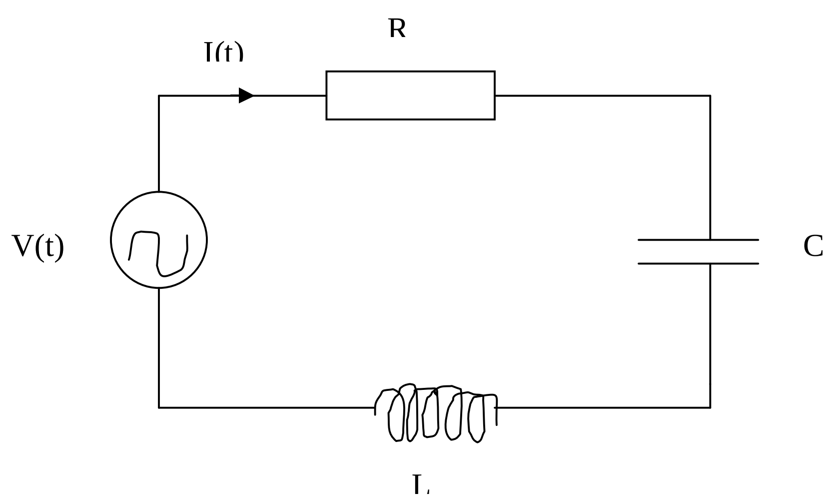
(ii) Hence describe briefly one physical application of these conditions (2mks)

d) In Quantum Mechanics, the probability density of a particle at a point is given by the product φ*φ where φ is the particle’s wave function and φ* is its complex conjugate. Determine the probability density of a particle whose wave function is

$\varphi = A\exp\left[i\left(\frac{8\Pi^2 mE}{h^2}\right)^{1/2} x\right]$ where A is a constant, I denotes the complex notation and all other symbols have their usual meanings. (3mks)

Question 4 (15 marks)

The diagram below shows a fixed resistor of a resistance R, an inductor of inductance L, a capacitor of capacitance C and an a.c. voltage source V(t) connected in series. The current in the circuit is I(t) and the charge on the capacitor plates is q(t).



a) Express the voltage equation of the circuit as a second order differential equation in q.

b) Given that R = 18Ω, C = 3.57x10-2F, L = 2H and V(t) = Sint and that q(t) = 0 and I(t) = 1A at t = 0, determine the expression for the instantaneous charge q(t) on the capacitor plates. (13mks)

Question 5 (15 marks)

a) State two conditions which must be satisfied by a differential equation of the form P(x,y)dx + Q(x,y)dy = 0 to be an exact differential equation, where P(x,y) and Q(x,y) are functions of x and y. (2mks)

b) A particle of mass 5gm oscillates along the x-axis about a fixed point on its path, acted on by a restoring force equal to 20x where x is the displacement from the fixed point. The particle was initially at rest at x = 5 units.

(i) Write down the differential equation of motion of the particle. (2mks)

(ii) Hence apply Laplace transform to determine an expression for the displacement x of the particle at any time t. (6mks)

c) Determine the eigen values of the matrix operator $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 2 & 4 \end{pmatrix}$ (5mks)