## COURSE TITLE: MATHEMATICS FOR ECONOMICS 1

## STREAM:

DAY:
TIME:
9.00-12.00 P.M

DATE:
8/12/2010

## INSTRUCTIONS:

1. Answer question ONE and any other TWO questions
2. Apart from question ONE; all other questions carry equal marks. Marks for subdivisions are shown in brackets.
3. Calculators are allowed in the examination room provided they are not programmable and can store or recall information.
4. Marks will be awarded to candidates who demonstrate clarity and accuracy of presentation.
5. Diagrams should be used where helpful.

## PLEASE TURNOVER

## QUESTION 1

a. i) What is the meaning of mathematical economics?
(2mks)
ii) Explain the advantages of mathematical economics in the study of economics
b. In a marketing survey of 100 randomly chosen students, a marketing questionnaire included the following three questions,

- Do you own a TV?
- Do you own a car?
- Do you own a TV and a car

75 answered yes to (i), 45 answered yes to (ii) and 35 answered yes to (iii).
i) How many students owned either a car or a TV?
ii) How many students did not own either a car or a TV?
iii) How many students owned a car but not a TV?
iv) How many students did not own both a car and a TV?
(Total 5 marks)
c. i) An individual's level of consumption is $30(\mathrm{C}=30)$ per month when his/her income equals zero i.e.

When $\mathrm{Y}=0$.
His/her consumption is 150 per month when his/her income is 150. Assuming a linear relationship between C and Y , determine the individual's consumption function
( 2.5 mks )
ii. At a price of 88 the quantity demanded of a good is 2000 and at price of $38 \mathrm{Q}_{\mathrm{d}}$ is 12000 .

Assuming a linear relationship between price and quantity demand, find a linear demand function that models the price demand relationship
d. The TC function of a product is related to the number of units of output $(Q)$ by the following equation,:
$T C=33+2 Q$
i. What is the break even level of output if price is fixed at 13 per unit? Sketch your results (3mks)
iii. Why would the producer not fix the price at 1 per unit?
e. Given the following total revenue function.
$\mathrm{TR}=20 \mathrm{Q}-0.02 \mathrm{Q}^{2}$
(i) Find the rate of change of TR function above.
(ii) Find the change in total revenue if Q changes from 100 to 400
f. Given the total cost function of the form,
$T C=10+0.06 Q+0.0001 Q^{2}$
i. Find the level of output $(\mathrm{Q})$ that minimizes ATC and minimizes ATC.
(3mks)
ii. Verify that when ATC is minimized, the S.O.C is satisfied

## QUESTION 2

a) Define a set and explain the two methods that are commonly used to represent elements of a set.
b) Assume that the direct tax (T) system takes $25 \%$ of income $(\mathrm{Y})$ regardless of the size and that an individual consumption (C) is 500 plus $80 \%$ of disposable income $\left(\mathrm{Y}^{\mathrm{d}}\right)$. Express the relation $\mathrm{C}=\mathrm{f}(\mathrm{Y})$ in algebra and in diagram
c) Suppose that the demand function for good X is
$\mathrm{Q}=6-0.2 \mathrm{P}_{\mathrm{x}}+\mathrm{M}+0.7 \mathrm{P}_{\mathrm{y}}$

Where:
$\mathrm{P}_{\mathrm{x}}=$ Price of good X
$\mathrm{M}=$ Consumers Income
$\mathrm{P}_{\mathrm{y}}=$ the price of related good Y .
i) Identify and distinguish between dependent and independent variable(s)
(3mks)
ii) Explain the relationship between goods X and Y
d) The Research Department in a company that manufactures AM/FM radios established the following price-demand function:

$$
\mathrm{P}=50-1.25 \mathrm{Q}
$$

i) By transforming the quadratic TR function into the following general form

$$
f(Q)=a(Q-h)^{2}+k
$$

Where : $a$, $h$ and $k=$ consatnts
Find the value of output ( Q ) that yields maximum total revenue and the value of maximum total revenue.
ii) Plot/graph your results in (i) above.

## QUESTION 3

a) A firm's $\mathrm{FC}=24,000$ and Variable costs per unit are 6.20 . The price per unit of output is 8.70. How many units of output should be sold for the firm to break even? Sketch your results.
(4mks)
b) Given the following equations,

$$
\begin{aligned}
& P=2 Q^{2}+10 Q+10 \\
& P=-Q^{2}-5 Q+52
\end{aligned}
$$

i. Identify which of the two functions is demand and supply function explaining your answer in each case
ii. Calculate the equilibrium price and quantity.
c) Given the following general market model: calculate the equilibrium prices $P_{1}$ and $P_{2}$, and equilibrium quantities $Q_{1}$ and $Q_{2}$.
(10mks)
$Q_{D 1}=3000-0.56 P_{1}+0.24 P_{2}$
$Q_{D 2}=6000+0.21 P_{1}-0.31 P_{2}$
$Q_{S 1}=-400+0.44 P_{1}-0.13 P_{2}$
$Q_{S 2}=-700-0.73 P_{1}+0.45 P_{2}$

## QUESTION 4

a) Find the rate of change of AC with respect to Q for the following average cost function

$$
\begin{equation*}
\mathrm{AC}=\mathrm{Q}^{2}+12 \mathrm{Q}+70 \tag{3mks}
\end{equation*}
$$

b) The total sales $S$ (in thousands) for a compact disk are given by:

$$
S(t)=\frac{90 t^{2}}{t^{2}+50}
$$

Where:
$t=$ Number of months since the release of the CD
(i) Find marginal sales per month.
(ii) Find total sales and marginal sales after 10 months.
(iii) Use the results in part (ii) to estimate the sales after 11months.
(iv) Find the exact total sales after 11 months and exact sales for the $11^{\text {th }}$ month.
(3mks)
c) Given the demand function of the form:
$P=-Q^{2}-4 Q+96$
Find and interpret price elasticity of demand when $\mathrm{P}=51$
(6mks)

## QUESTION 5

a) A monopolist demand and cost functions are given by:

$$
\begin{aligned}
& P=30-0.75 Q \\
& A C-\frac{30}{Q}=9+0.3 Q
\end{aligned}
$$

i. Find level of Q that maximizes $\operatorname{profit}(\pi)$ and maximum $\pi$.
ii. Verify that when $\pi$ are maximized, $\mathrm{MR}=\mathrm{MC}$.
(2mks)
iii. Suppose a tax rate of 8.4 per unit of output is imposed on the monopolist. Find the new Q that will maximize $\pi$
b) The supply and demand equations of a good are given by:-

$$
\begin{aligned}
& P=Q_{s}+8 \\
& P=-3 Q_{d}+80
\end{aligned}
$$

If the government decides to impose a tax ' $t$ ' per unit, find;
i) Value of tax which maximizes tax revenue and maximum tax revenue.
ii) Equilibrium price and quantity
iii) Price received by suppliers.

