

UNIVERSITY

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## 2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

## COURSE CODE: ECON 121

COURSE TITLE: MATHEMATICS FOR ECONOMISTS I
STREAM: Y1S2

## DAY: THURSDAY

TIME:
8.30 - 10.30 A.M.

DATE:
7/8/2008

## INSTRUCTIONS:

1. Answer QUESTION ONE and any other TWO questions.

PLEASE TURN OVER

## QUESTION ONE

(a) Given the sets:

$$
\begin{aligned}
& A=\{1,3,5,7\} \\
& B=\{1,2,3,4,5,6,7,8,9,10\} \\
& C=\{2,4,6,8\}
\end{aligned}
$$

## Find:

(i) $\mathrm{A} \cap \mathrm{B}$
(ii) AYC
(iii) If in (a) above $u$ (universal set) $=\left\{\frac{x}{x}\right.$ is a positive integer $\}$ Find $\mathrm{A} \cap \mathrm{A}{ }^{\prime}$
(3mks)
(b) Solve the following inequalities:
(i) $-2 \mathrm{x}+1 \leq \mathrm{x} \leq 6-\mathrm{x}$
(2mks)
(ii) $|x-4|>6$
(2mks)
(c) The demand for the product of a firm varies with the price that the firm charges for the product. The firm estimates that annual total revenue (R) is a function of the price $(\mathrm{P})$ specifically,

$$
R=f(p)=-50 P^{2}+500 p
$$

(i) Determine the price which should be charged in order to maximize total revenue?
(ii) What is the maximum value of annual total revenue?
(d) The total cost of producing $q$ units of a certain product is described by the function.

$$
C=100,000+1500 q+0.2 q^{2}
$$

Where C is the total cost stated in shillings; Determine the number of units of q that should be produced in order to minimize the average cost per unit.
(e) (i) Without using tables show that:

$$
\begin{equation*}
1 / 2 \log \frac{25}{9}-\log \frac{15}{4}+\frac{2}{3} \log \frac{27}{8}=0 \tag{3mks}
\end{equation*}
$$

(ii) Show that $\log _{\mathrm{a}} \mathrm{b}=\frac{1}{\log _{b} a}$

## OUESTION TWO

A consumer's utility function is given by $\mathrm{U}=\mathrm{q}_{1} \mathrm{q}_{2}$ where $\mathrm{q}_{1}$ and $\mathrm{q}_{2}$ are the quantities of the two commodities consumed. If the price of $q_{1}$ is Shs. 6 and that of $q_{2}$ is Shs. 3 and the budget is Shs60,
(i) Write out a constrained utility maximization problem out of the information given.
(ii) What is the corresponding augmented objective function?
(iii) Find the levels of $\mathrm{q}_{1}$ and $\mathrm{q}_{2}$ that will satisfy the first order condition for maximum
(iii) Compute the optimum value of U
(20mks)

## QUESTION THREE

The demand and total cost function for a firm are given by;

$$
\begin{aligned}
& \mathrm{P}=7-\frac{2}{5} \mathrm{Q} \\
& \mathrm{TC}=\frac{4}{7} \mathrm{Q}^{3}-\frac{3}{4} \mathrm{Q}^{2}+7 \mathrm{Q}+5
\end{aligned}
$$

Find:
(i) The level of Q and P that will maximize profits.
(ii) The level of Q that will maximize TR
(iii) The level of Q that will minimize AVC
(iv) The level of Q that will minimize MC
(v) The minimum AVC and MC
(20mks)

## QUESTION FOUR

(a) Consider the following two commodity market model:

$$
\begin{aligned}
& \mathrm{Q}_{\mathrm{d} 1}=8-2 \mathrm{P}_{1}+\mathrm{P}_{2} \\
& \mathrm{Q}_{\mathrm{s} 1}=-5+3 \mathrm{P}_{1} \\
& \mathrm{Q}_{\mathrm{d} 2}=16+\mathrm{P}_{1}-\mathrm{P}_{2} \\
& \mathrm{Q}_{\mathrm{s} 2}=-1+2 \mathrm{P}_{2}
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{Q}_{\mathrm{d} 1}=\mathrm{Q}_{\mathrm{s} 1} \\
& \mathrm{Q}_{\mathrm{d} 2}=\mathrm{Q}_{\mathrm{s} 2}
\end{aligned}
$$

Determine by Cramer's rule:
(i) The value of equilibrium prices

## (3mks)

(ii) The value of equilibrium quantities
(b) Consider the following national income model for a closed economy:

$$
\begin{aligned}
& \mathrm{Y}=\mathrm{C}+\mathrm{I}_{0}+\mathrm{G}_{0} \\
& \mathrm{C}=\mathrm{c}_{0}+\mathrm{c}_{1} \mathrm{Y}
\end{aligned}
$$

Where:
Y and C are endogenous variables, $\mathrm{I}_{0}$ and $\mathrm{G}_{0}$ are the exogenous variables and $\mathrm{C}_{0}$ and $\mathrm{C}_{1}$ are the parameters.

## Required:

Determine by Cramer's rule the equilibrium $\bar{Y}$ and $\bar{C}$
(c) Solve the following equations by matrix inverse method

$$
\begin{aligned}
& x_{1}+2 x_{2}+3 x_{3}=3 \\
& 2 x_{1}+4 x_{2}+5 x_{3}=4 \\
& 3 x_{1}+5 x_{2}+6 x_{3}=8
\end{aligned}
$$

(9mks)

## OUESTION FIVE

(a) Find the critical value of $z$ given

$$
\begin{aligned}
& Z=x+2 x y+y \\
& \text { Subject to: } x+y-4=0
\end{aligned}
$$

(b) Determine $\frac{d y}{d x}$ given that

$$
Y=\left(2 x^{3}+x^{2}-3 x+18\right)^{5}
$$

(c) Find the following:
(i) $\quad \int-1 / 4 x^{-3} d x$
(ii) $\quad \int\left(x^{3 / 4}+\frac{3}{7} x^{-1 / 2}+x^{5}\right) d x$

