KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/2010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE

ECONOMICS AND MATHEMATICS

COURSE CODE: ECON 211

COURSE TITLE: MATHEMATICS FOR ECONOMISTS II

STREAM: Y3S1

DAY: FRIDAY

TIME: 9.00 -11.00 A.M.

DATE: 13/08/2010

INSTRUCTIONS

1. Answer Question **ONE** and any other **TWO** questions.
2. Apart from question **ONE;** all other questions carry equal marks. Marks for subdivisions are shown in brackets.
3. Calculators are allowed in the examination room provided they are not programmable and can store or recall information.
4. Marks will be awarded to candidates who demonstrate clarity and accuracy of presentation.
5. Diagrams should be used where necessary

PLEASE TURN OVER

QUESTION ONE

a) Given the following commodity market model,

$$Q_D = a_0 - a_1 P$$

$$Q_S = -b_0 + b_1 P$$

Find the effects of:

i. Changes in b_0 and b_1 on equilibrium price. **(3marks)**

ii. Changes in a_0 and a_1 on equilibrium price. **(3marks)**

b) For the following utility function,

$$U = Q_1^{\frac{1}{2}} Q_2^{\frac{5}{2}}$$

i. Determine the Marginal Utilities of Q_1 and Q_2 (3mks)

ii. Find out whether the Utility function displays the characteristics of diminishing or increasing marginal utility with respect to Q_1 and Q_2. **(4marks)**

c) Utility function is given by,

$$U = Q_1^2 Q_2^3 Q_3^4$$

If Q_1 changes from I to 2, Q_2 changes from 2 to 4 and Q_3 changes from 3 to 5, find the total change in utility. **(5marks)**

d) A perfectly competitive firm produces and sells two goods Q_1 and Q_2 at prices 1000 and 800 per unit respectively. The total joint cost function of producing the two goods is given by,

$$TC = 2Q_1^2 + 2Q_1 Q_2 + Q_2^2$$

Find,

i. Levels of Q_1 and Q_2 that maximizes profit. **(6marks)**

ii. Maximum level of profit **(2marks)**

iii. Verify that the second and third order conditions are satisfied **(4marks)**

QUESTION TWO

a) A firm's production function is given by;

$$Q = 2K^{\frac{1}{2}}L^{\frac{1}{2}}$$

The price of labor and capital are 3 and 4 respectively.

i. Using substitution method, find the values of labor and capital which minimizes total input cost if the firm is contracted to provide 160 units of output. **(6marks)**

ii. Verify that when the firm is employing units of capital and labor that minimizes total cost, the ratio of marginal physical products of factor inputs and factor prices are equal. That is $\frac{MPP_L}{MPP_K} = \frac{P_L}{P_K}$ **(4marks)**

b) A monopolistic producer of two goods has the following joint total cost function and demand functions for the two goods,

$$TC = 5Q_1 + 10Q_2$$

$$P_1 = 50 - Q_1 - Q_2$$

$$P_2 = 100 - Q_1 - 4Q_2$$

If total cost are fixed at 100, using Lagrange multiplier method find,

i. Level of Q_1 and Q_2 that maximizes profit. **(6marks)**

ii. Maximum value of profits. **(2marks)**

iii. Estimate the new level of optimal profit if total cost rises to 101. **(2marks)**

QUESTION THREE

a) Given the marginal revenue function of the form,

$$MR = 45 - Q$$

i. Find the Total Revenue function (TR). **(4marks)**

ii. Deduce the corresponding demand function. **(2marks)**

b) The monthly sales of a particular personal computer are expected to decline at the rate of,

$S'(t) = -25t^{\frac{2}{3}}$ Computers per month

Where, t is the time in months and S(t) is the number computers sold each month. The company plans to stop manufacturing this computer when the monthly sales reach 800 computers.

i. If the monthly sales now (t=0) are 2,000 find S(t). **(3marks)**

ii. How long will the company continue to manufacture this computer? **(3marks)**

c) Find the consumers' and the producers' surplus at the equilibrium price level for the following Price-demand and price supply equations.

$$P = 20 - 0.05Q$$

(8marks)

$$P = 2 + 0.0002Q^2$$

QUESTION FOUR

a) The equilibrium prices P_1 and P_2 for two goods satisfy the following equations,

$$9P_1 + P_2 = 43$$

$$2P_1 + 7P_2 = 57$$

i. Express this system in matrix form. **(2marks)**

ii. Solve for P_1 and P_2 by using matrix inverse. **(5marks)**

b) An economy is based on three sectors, Coal, Oil and Transportation. Production of a shilling worth of Coal requires an input of 20cts from the Coal sector and 40cts from the transportation sector. Production of a shilling worth of Oil requires an input of 10cts from the Oil sector and 20cts from the transportation sector. Production of a shilling worth of transportation requires an input of 40cts from the Coal sector, 20cts from the Oil sector and 20cts from the transportation sector.

Find the output from each sector that is needed to satisfy a final demand of sh.30billions for Coal, sh.10 billions for Oil, and sh.20billions for transportation. **(13marks)**

QUESTION FIVE

a) Define linear programming (LP). **(3marks)**

b) Highlight the general strategies of formulating an L.P problem. **(4marks)**

c) A food producer uses two plants, P_1 and P_2. After processing, beef is graded into high, medium and low quality foodstuff. High quality beef is sold to butchers, medium quality beef is used in supermarket ready meals and the low quality beef is used in dog food. The producer is contracted to provide 120kgs of high quality beef, 80kgs of medium quality beef, and 240kgs of low quality beef each week. It costs sh.4000 per day to run plant P_1 and sh.3200 per day to run plant P_2. Each day P_1 processes 60kgs of high quality beef, 20kgs of medium quality beef, and 40kgs of low quality beef. The corresponding quantities for P_2 are 20kgs, 20kgs, and 120kgs respectively. How many days each week should the plants be operated to fulfill the beef contract most economically? **(13marks)**