

# FOR THE DEGREE OF BACHELOR OF ECONOMICS AND MATHEMATICS 

COURSE CODE: ECON 211
COURSE TITLE: MATHEMATICS FOR ECONOMISTS II
STREAM: Y2S1
DAY:
MONDAY
TIME:
11.00-1.00 P.M.

DATE:
8/12/2008

## INSTRUCTIONS:

1. Answer question ONE and any other TWO questions

PLEASE TURN OVER

1. a) Rafiki Ltd is drawing up production plans for the coming year. Four products are available with the following financial characteristics:

|  | A | B | C | D |
| :--- | :--- | :--- | :--- | :--- |
| Amounts per unit: |  |  |  |  |
| Selling price (Shs) | 55 | 53 | 97 | 86 |
| Cost of materials (Shs) | 17 | 25 | 19 | 11 |
| Labour hours - Grade A | 10 | 6 | - | - |
| $\quad$ Grade B | - | - | 10 | 20 |
| Grade C | - | - | 12 | 6 |
| Variable Overheads | 6 | 7 | 5 | 6 |

Fixed overheads of the firm amount to 35,500 per annum. Each grade of labour is paid Shs. 15 per hour but skills are specific to a grade so that an employee in one grade cannot be used to undertake the work of another grade. The annual supply of each grade is limited to the following maximum: grade A, 9,000 hours, grade B, 14,500 hours, grade C, 12,000 hours. There is no effective limitation on the volume of sales of any product.

## Required:

(i) Calculate the product mix which will maximize profit for the year and state the amount of profit. (Use simplex method).
(14marks)
(ii) Calculate the minimum price at which the sale of product A would be worth while.
(2 marks)
(iii) Calculate the amount by which profit could be increased if the supply of grade A labour were increased by one hour.
(a) Multi Band Enterprises manufacturer's two products, a portable radio (PR) and a citizens band (CB) radio. The marketing manager states "we can sell all that can be produced in the near future". She then asks the operations manager, "What is your production capacity per month?" The operations manager replies that his output capacity depends on which product is produced.
"Three kinds of labour are required for making our products: subassembly, Assembly and inspection labour. The two products require different amounts of each kind of labour, so our capacity for next month depends on which products we produce. Next month we will have 316 hours of subassembly labour available, 354 hours of Assembly labour, and 62 hours of inspection labour."

The operations manager knows that each CB radio requires 0.4 hours of subassembly labour time. 0.5 hours of assembly labour 0.10 hours of inspection labour time. A portable radio can be produced using 0.5 hours of subassembly labour, 0.3 hours of assembly labour and 0.10 hours of inspection labour time.

The Deputy Director says, "We know that each CB that we produce and sell contributes Ksh. 50 towards profit. Each PR has a Sh. 40 contribution margin."

## Required:

Formulate the above as a linear programming model and set-up the initial simplex tableau only for the model.
2. a) The demand for a commodity is given by;

$$
P=400-q
$$

The average cost of producing the commodity is given by:

$$
A T C=\frac{1000}{q}+100-5 q+q^{2}
$$

Where P is the price in shillings and q is the quantity in kilogrammes.

## Required:

(i) What does $\underline{1000}$ in the ATC equation represent economically? (2 marks) q
(ii) Determine the output that leads to maximum profit and the profit at that level of output.
(8 marks)
(b) Alpha industries sell two products, x and y in related markets, with demand functions given by

$$
P_{x}-13+2 x+y=0
$$

$$
P_{y}-13+x+2 y=0
$$

The total cost, in shillings, is given by:

$$
\mathrm{TC}=\mathrm{x}+\mathrm{y}
$$

## Required:

Determine the price and the output for each good which will maximize profits.
(10 marks)
3. a) Evaluate the determinant of the following matrix.

$$
\left[\begin{array}{cccc}
1 & -2 & -3 & 4  \tag{5marks}\\
-2 & 3 & 4 & -5 \\
3 & -4 & -5 & 6 \\
-4 & 5 & 6 & -7
\end{array}\right]
$$

(b) If $A=\left[\begin{array}{rr}2 & -1 \\ 3 & 2 \\ 4 & -6\end{array}\right]$
(i) Find a matrix I (identify) such that A.I = A
(3 marks)
(ii) Is it true that I.A $=\mathrm{A}$ in this example?
(2 marks)
c) Solve the following system of equations using the inverse method.

$$
\begin{aligned}
& x_{1}+2 x_{2}=5 \\
& x_{1}-x_{3}=-15 \\
& -x_{1}+3 x_{2}+2 x_{3}=40
\end{aligned}
$$

(10 Marks)
4. (a) The function describing the marginal cost of producing a product is $\mathrm{MC}=\mathrm{x}+100$
Where $x$ equals the number of unit produce. It is also known that total cost equals Ksh. 40,000 when $\mathrm{x}=100$.

Determine (i) The fixed cost
(ii) The total cost function
(3 marks)
(4 marks)
(b) Evaluate

$$
\int(5 x-3)^{3}(5) \mathrm{d} x
$$

(5 marks)
(c ) Solve the following first order linear differential equations

$$
\begin{array}{ll}
\text { i) } & \frac{d y}{d t}+2 t y=t  \tag{4marks}\\
\text { ii) } & \frac{d y}{d t}+4 t y=4 t
\end{array}
$$

(4 marks)
5. (a) Consider an economy with three industries: coal, electricity, railways. To produce Sh.1of coal requires Sh. 0.25 worth of electricity and sh. 0.25 rail costs for transportation. To produce Sh. 1 of electricity requires sh. 0.65 worth of coal for fuel, sh. 0.05 of electricity for the auxiliary equipment, and sh. 0.05 for transport. To provide sh. 1 worth of transport, the railway requires sh. 0.55 coal for fuel and sh.0.10 electricity. Each week the external demand for coal is sh. 50.000 and the external demand for electricity is $\operatorname{sh} .25,000$. There is no external demand for the railway. What should be the weekly production schedule for each industry?
(14 marks)
(b) Explain briefly what you understand by each of the following terms as used in input/output analysis:
(i) Technical coefficient
(ii) Value Added
(iii) Closed 1/0 Model

