KABARAK   **UNIVERSITY**

EXAMINATIONS
2008/2009 ACADEMIC YEAR
FOR THE DEGREE OF BACHELOR OF SCIENCE IN
ENVIRONMENTAL SCIENCE

COURSE CODE: **ENVS 112**

COURSE TITLE: **MATHEMATICS FOR SCIENTISTS**

STREAM: **Y1S1**

DAY: **FRIDAY**

TIME: **2.00 - 5.00 P.M.**

DATE: **07/08/2009**

INSTRUCTIONS:

Answer question **ONE** is **(compulsory)** and any other **Three** questions.

PLEASE TURN OVER

QUESTION 1 (COMPULSORY) 40 MARKS

a) Differentiate the following functions with respect to x

    i) $f(x)=\sin 3x$ **(2 marks)**

    ii) $y = 4xe^{6x-5}$ **(3 marks)**

b) Evaluate the following

    i) $\int_{1}^{2}(4x + 7x^2 + 6)dx$ **(2 marks)**

    ii) $\int \dfrac{x^2 + 7x + 19}{x^2}dx$ **(2 marks)**

c)   Solve the following linear equation using row reduction
        2x + y + z = 4
        x -3y+z= -1
        x + y + z = 3

                                                                   (4 marks)

d) Convert the following decimal numbers to binary equivalent

    i) 18   ii) 23   **(4 marks)**

e) i) Show that $\sim (p \wedge q) = p \overline{\wedge} q$ **(2marks)**
     ii) Show that $p \overline{\wedge} q = \sim p \vee \sim q$

f) What is the coefficient of $x^3 y^7$ in

        i) $(x + y)^{10}$ **(2 marks)**
        ii) $(2x - 9y)^{10}$ **(2 marks)**

g) Given the following function f(x) = 4x+8 and $g(x) = e^{5x+4}$, find
      i) f(0) , f (4), f(g(x))   **(3 marks)**
      ii) Find the inverse function of f(x) and g(x)   **(2 marks)**

h) Evaluate

    i) $\displaystyle\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3}$ **(2marks)**

    ii) $\displaystyle\lim_{x \to 2} (4x+8)$ **(1mark)**

j) The performance of students in KCSE at Mabao high school is summarized below

Grade	A	B	C
Academy	20	32	23
Government school	34	21	12
Private	71	34	40

A student is selected at random. Find the probability that

      i) He got grade A in a government school   **(2 marks)**

      ii) He got Grade A or B   **(2 marks)**

k) The expenditure on food by a sample 700 households selected at random in Nairobi West is given below.

Expenditure	No of House holds
4000- 5000	150
5000 -6000	240
6000- 7000	130
7000- 10000	113
10000-20,000	67

   i) Estimate the mean expenditure for the population   **(2 marks)**

   ii) Estimate the variance of the expenditure   **(3marks)**

QUESTION 2 (20 MARKS)

(a)   A market researcher investigating consumers' preference for three brands of beverages namely; coffee, tea and cocoa, in Ongata town gathered the following information:

From a sample of 800 consumers, 230 took coffee, 245 took tea and 325 took cocoa, 30 took all the three beverages, 70 took coffee and cocoa, 110 took coffee only, 185 took cocoa only.

Required:

(i)   Present the above information in a Venn diagram.   **(4 marks)**

(ii)   The number of customers who took tea only.   **(2 marks)**

(iii)   The number of customers who took coffee and tea only.   **(2 marks)**

(iv)   The number of customers who took tea and cocoa only.   **(2 marks)**

(v)   The number of customers who took none of the beverages.   **(2 marks)**

b) The height s in m of an object fired straight up from the ground with initial velocity of 200m/s is given by

$$S = -16t^2 + 200t$$

Where t is the time in seconds

   i)   How fast is the object moving after 1 second   **(2 marks)**

   ii) During which interval of time is the speed decreasing   **(2 marks)**

   iii) Which interval is the speed increasing   **(2 marks)**

   iv) Find the maximum height   **(2 marks)**

QUESTION 3 (20 MARKS)

(a) A company has three branches (N, M and Q) that is used in the manufacture of its
  commodity.
  These components are shipped for export in the US. The proportion of defective items is
  given as follows

Components	Quantity	defective
N	30,000	2%
M	24000	5%
Q	46,000	3%
	100,000	

Using a probability tree find
  i)   Probability of selecting a defective item   **(3 marks)**
  ii)   probability of selecting at least a good item   **(3 marks)**
  iii)   An item exported to the US is found to be defective. What is the probability that it
        was produced by Branch Q.   **(4 marks)**

(b) A company has three branches for manufacturing three items A, B and C. The branches
  are Kisumu, Nairobi and Mombasa.   The matrix N shown below indicates the number
  of each type of item produced

$$
N = \begin{pmatrix} A & B & C \\ 150 & 300 & 200 \\ 100 & 100 & 150 \\ 120 & 400 & 300 \end{pmatrix} \begin{matrix} \text{Kisumu} \\ \text{Nairobi} \\ \text{Mombasa} \end{matrix}
$$

The items are sold at the prices given below

$$
\begin{pmatrix} A & B & C \\ 10 & 8 & 8 \\ 12 & 6 & 6 \\ 8 & 4 & 5 \end{pmatrix} \begin{matrix} \text{Kisumu} \\ \text{Nairobi} \\ \text{Mombasa} \end{matrix}
$$

i) Find the product of matrix N and Row matrix (1 1 1)   **(2 marks)**
   State the significance of the product   **(2 marks)**
ii) Find the transpose of matrix P   **(2 marks)**
iii) Using matrix multiplication find the total revenue earned from the sales of the products in
   Nairobi   **(4 marks)**

QUESTION 4 (20 MARKS)

 (a)   An electric utility company has found that the weekly number of occurrences of lightning striking the transformers has a poisson distribution with mean 0.4.

Required:
     (i)   The probability that no transformer will be struck in a week.   **(3 marks)**
     (ii)   The probability that at most two transformers will be struck in a week.

(3 marks)

b)   The weight of students in a class has a normal distribution with a mean of 65kg and standard deviation of 7kg. A student is selected at random.

  Calculate
     i) Probability that the weight of a student is less than 77 kg   **(2 marks)**
     ii) The probability that the weight of a student lies in the range 34-77kg   **(2marks)**
     iii) 24 % of the students are considered overweight. What is the minimum weight for one to be considered overweight   **(3 marks)**

c) Us the binomial theorem to prove that

$$3^n = \sum_{r=0} C(n,r)2^r$$
     (3 marks)

d) Expand $f(x) = (1+x)^4$ up to the term with x^4 . Hence estimate 1.05^4 **(4 marks)**

QUESTION 5 (20 MARKS)

a) Using Crammers rule, solve for x, y and z in the following set of equations

     x+y-z=2

     x+z=1

     x-y-z=0   **(6 marks)**

b) Consider the following propositions
     P: David is playing pool
     Q: David is inside
     R: David is doing his homework
     S: David is listening to music

Translate the following sentences into symbolic notation using p, q, r, s~, ∧ ,∨ and parentheses
     i)   Either David is playing pool or is inside   **(1mark)**
     ii)   David is playing pool and not doing his homework.   **(1mark)**
     iii)   David is not listening to music, nor his he playing pool, neither his he doing his homework   **(1mark)**
     iv)   David is inside doing his homework while listening to his music, and he is not playing pool.

(1 mark)

c) Construct a truth table for the following:

$[(p \vee q) \wedge (\sim r)] \leftrightarrow q$ **(6 marks)**

d) i) $\log_2 200 + \log_2 0.32$ **(2 marks)**

ii) Solve the equation

$\log_5(x+1) + \log_5(x-1) = 3$ **(2 marks)**

QUESTION 6 (20 MARKS)

a) A committee is to be chosen from a set of 9 women and 5 men. How many ways are there to form the committte if the committee has

i) 6 people, 3 women and 3men **(2marks)**

ii) 6 people including Mr. A **(3marks)**

iii) 6 people and atleast three women **(3marks)**

b) Plot a less than cumulative frequency curve for the following distribution for the data below

Income'000	**No of workers**
10-20	8
20-30	15
30-40	22
40-50	10
50-60	5

From the cumulative frequency curve, determine

i) The number of workers who earn less than Ksh35,000 **(3marks)**

ii) The number of workers who earn more than Ksh 25,000. Show it clearly on the graph.

(3marks)

c) In how many ways can 14 men be partitioned into 6 teams where the first team has 3 members, the second team has 3 members while the rest of the teams have two members each

(2 marks)

d) Solve $4^{3x+4} = 73$ **(4 marks)**