KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2010/2011 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS

COURSE CODE: MATH 425

COURSE TITLE: MULTIVARIATE ANALYSIS

STREAM: Y4S2

DAY: THURSDAY

TIME: 9.00 – 11.00 A.M.

DATE: 09/12/2009

INSTRUCTIONS:

1. Answer **QUESTION ONE** and **TWO** other questions
2. Show all your working method and be neat

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

a) Given **$\underline{X}$**=(x_1, x_2, x_3)´ where $f(\underline{x}) = e^{-x_2}(x_1 + x_3)$, 0< x_1< 1, 0 < x_3 < 1, x_2 > 0
find the mean vextor of **$\underline{X}$** (or alternatively find marginal means of x_1, x_2 and x_3) **[5 marks]**

b) Given that $\underline{X}$ ~ Np(μ, ∑) where $\underline{\mu} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ and $\sum = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 5 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

find the conditional distribution of $\underline{x_1} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ given X_3=x_3 **[10 marks]**

c) Derive expressions for the mean and variances of the following linear combinations in terms of the means and covariances of the random variables X_1, X_2, and X_3. **[15 marks]**

i) X_1 -2X_2
ii) -X_1 +2X_3
iii) X_1 +X_2 +X_3
iv) X_1 +2X_2 -X_3
v) 3X_1 -4X_2 if X_1 and X_2 are independent random variables.

QUESTION TWO (20 MARKS)

A) Let **x** = (x_1, x_2, x_3)´ be a random vector with a mean vector $\underline{\mu}$=(μ_1, μ_2, μ_3) and dispersion matrix S where μ and S are unknown. Given the values of a data matrix for a random sample of size n = 5 from **x**

$$X = \begin{bmatrix} 5 & 4 & 3 \\ 6 & 5 & 4 \\ 8 & 7 & 5 \\ 7 & 8 & 3 \\ 9 & 6 & 5 \end{bmatrix}$$

Compute
i) The sample mean vector **[2 marks]**
ii) Sample covariance matrix S and the unbiased estimate of Sn **[3 marks]**
iii) Sample correlation matrix **[3 marks]**
iv) State whether x_1 x_2, x_3 are independent from each other **[2 marks]**

B) Let **X** have covariance matrix **[10 marks]**

$$\sum = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 0 & 9 \end{bmatrix}$$

i) Determine **ρ** and $V^{1/2}$
ii) Multiply your matrices to check the relation $V^{1/2}$ **ρ** $V^{1/2}$ **=∑**

QUESTION THREE (20 MARKS)

A) Let X be $N_3(\mu, \Sigma)$ with $\mu'=(-2.5, 1.5, 4)$ and

$$\Sigma = \begin{bmatrix} 1 & -3 & 0 \\ -3 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Which of the following random variables are independent? Explain. **[10 marks]**

a) X_1 and X_2

b) X_2 and X_3

c) (X_1, X_2) and X_3

d) $\frac{X_1 + X_2}{2}$ *and* X_3

e) X_2 and $X_2 - \frac{5}{2}X_1 - X_3$

b) Let

$$\Sigma = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Determine the principal components Y1, Y2 and Y3. What can you say about the eigenvectors (and principal components) associated with eigenvalues that are not distinct? **[10 marks]**

QUESTION FOUR (20 MARKS)

A) Calculate the least square estimates $\hat{\beta}$, the residuals $\hat{\varepsilon}$, and the residual sum of squares for a straight-line model

$Y_i = \beta_0 + \beta_1 z_{j1} + \varepsilon_j$ fit to data **[10 marks]**

z_1	0	1	2	3	4
y	1	4	3	8	9

B) A researcher considered five companies, x_1, x_2, x_3, x_4, x_5 of Allied Chemical, DupPont, Union Carbide, Exxon and Texaco for weekly rates of returns respectively. The means and correlation matrix, **R** are given below:

$$\bar{x} = \begin{bmatrix} 0.0054 \\ 0.0048 \\ 0.0057 \\ 0.0063 \\ 0.0037 \end{bmatrix} \text{ and } R = \begin{bmatrix} 1.000 & 0.577 & 0.509 & 0.387 & 0.462 \\ 0.577 & 1.000 & 0.599 & 0.389 & 0.322 \\ 0.509 & 0.599 & 1.000 & 0.436 & 0.426 \\ 0.387 & 0.389 & 0.436 & 1.000 & 0.523 \\ 0.462 & 0.322 & 0.426 & 0.523 & 1.000 \end{bmatrix}$$

The eigenvalues and corresponding normalized eigenvectors of R were determined by a computer and are given below

$\hat{\lambda}_1 = 2.857, \hat{\gamma}_1 = [0.464, 0.457, 0.470, 0.421, 0.421]$

$\hat{\lambda}_2 = 0.809, \hat{\gamma}_2 = [0.240, 0.509, 0.260, -0.526, -0.582]$

$\lambda_3 = 0.540, \hat{\gamma}_4 = [-0.612, 0.178, 0.335, -0.541, -0.435]$

$\hat{\lambda}_4 = 0.452, \hat{\gamma}_4 = [0.387, 0.206, -0.662, 0.472, -0.382]$

$\hat{\lambda}_5 = 0.343, \hat{\gamma}_5 = [-0.451, 0.676, -0.400, -0.176, 0.385]$

i) Write down principal components that accounts for the communality of at least 73% of variations. **[5 marks]**

ii) Interpret the results you have stated in (i) as fully as possible **[5 marks]**