KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/2010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF COMPUTER SCIENCE

COURSE CODE: MATH 314

COURSE TITLE: NUMERICAL ANALYSIS

STREAM: Y3S1

DAY: FRIDAY

TIME: 2.00 – 4.00 P.M.

DATE: 06/08/2010

INSTRUCTIONS:

- Answer Question **ONE** and any other **TWO** questions

PLEASE TURNOVER

QUESTION ONE: 30 MARKS

a) Define the following terms as used in connection with errors.

(i) Relative error

(ii) Round-off error (2 marks)

b) Estimate the error in evaluating $f(x, y, z) = x^2 + \frac{y}{z}$ for $x = 1.23, y = 2.34, z = 3.45$

where each of these is assumed to be correctly rounded to the number of significant figures shown. Hence find f. (5 marks)

c) Construct a difference table for the values shown in the table below and find $f(x)$

x	0	1	2	3	4	5	6	7
$f(x)$	1	2	4	7	11	16	22	29

(5 marks)

d) Use the Lagrange interpolation polynomial to find the equation of the curve passing through the points

x:	0	1	2
f(x):	0	1	20

(5 marks)

e) Use the iterative method to solve the equation $x^3 - x^2 - 1 = 0$ (6 Marks)

f) Set-up a New- Raphson formula to find the cube root of N where N is a positive number and hence find the cube root of 15. (7 marks)

QUESTION TWO: 20 MARKS

a) Consider the sequence of values $f(x) = (0, 0, 0, \varepsilon, 0, 0, 0)$ where ε is an error. Show that

(i) Error spreads and increases in magnitude as the order of the difference is increased.

(ii) The error in each column have binomial coefficients. (6 marks)

b) Prove that $\sum_{k}^{n-1} \Delta y_k = y_n - y_0$ (4 marks)

c) (i) the new forward formula for the collocation polynomial can be written as

$p_k = \sum_{i=0}^{n} \binom{k}{i} \Delta^i y_0$. Prove that $y_1 = y_0 + \Delta y_0, y_2 = y_0 + 2\Delta y_0 + \Delta^2 y_{0,}$

$y_3 = y_0 + 3\Delta y_0 + 3\Delta^2 y_0 + \Delta^3 y_0$.

(ii) Apply Newton's formula to find a polynomial of degree four or less which takes the values below.

x_k:	1	2	3	4	5
y_k:	1	-1	1	-1	1

(10 marks)

QUESTION THREE: 20 MARKS

a) Show that the equation $f(x) = x^3 - x - 1 = 0$ has o root in the interval $[0,1]$. Apply Newton-Raphson method to determine the root of the equation. (5 marks)

b) Derive the Newton's backwards interpolation formulae. Hence or otherwise obtain the collocation polynomial for the following data:

x:	0.1	0.2	0.3	0.4	0.5
f(x):	1.40	1.56	1.76	2.00	2.28

Interpolate at x=0.25. (8 marks)

c) Let $\bar{x}$ and $\bar{y}$ be approximate numbers with errors ε_1 and ε_2 respectively to the exact numbers x and y. if $f = \frac{x}{y}$ determine the maximum error in computing f and further find f as accurately as possible if $x = 3.55 \pm 0.05, y = 3.75 \pm 0.05$ (7 marks)

QUESTION FOUR: 20 MARKS

a) Find the first and the second derivative of the function tabulated below at $x = 0.6$

x:	0.4	0.5	0.6	0.7	0.8
y:	1.5838	1.7974	2.0442	2.3275	2.6511 (10 marks)

b) Use Lagrange's interpolation formula to find y(9.5)given

x:	7	8	9	19	
y:	3	1	1	9	(10 marks)

QUESTION FIVE: 20 MARKS

a) Evaluate $I = \leftrightarrow \int_{1.00}^{1.30} \sqrt{x}dx$ with subdivision of 0.05 using

(i) Trapezoidal rule

(ii) Simpson's $\frac{1}{3}$ rule

(iii) Direct integration [exact solution]. Estimate the error involved in each case.(12 marks)

b) Using Newton's divided difference formula find f(x) and hence f(6) from the following table.

x;	1	2	7	8	
y:	1	5	5	4	(8 marks)