

KABARAK



UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

**FOR THE DEGREE OF BACHELOR OF EDUCATION
SCIENCE**

COURSE CODE: MATH 314

COURSE TITLE: NUMERICAL ANALYSIS EXAM

STREAM: SESSION VI & VII

DAY: MONDAY

TIME: 2.00 – 4.00 P.M.

DATE: 06/04/2009

INSTRUCTIONS TO CANDIDATES:

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) Find the relative error of the number 8.6 given that both of its digits are correct. (2 marks)

(b) Use Gauss's interpolation formula to get y_{16} given

x:	5	10	15	20	25
y:	26.782	19.951	14.001	8.762	4.163

(c) Find and correct the error in the sequence of numbers below:

1, 3, 11, 31, 69, 113, 223, 351, 521, 739 (7 marks)

(d) Show that $\Delta^3 y_k = y_{k+3} - 3y_{k+2} + 3y_{k+1} - y_k$ (3 marks)

(e) Given that $y = x^3 + x^2 - 2x + 1$, Determine the values of y for $0 \leq x \leq 5$ and form a difference table. Determine the value of y at $x = 6$ by extending the table and verify that the same value is obtained by substitution. (5 marks)

(f) Find the value of $\sqrt{20}$ correct to 3 decimal places by using the iterative formula

$$x_{i+1} = \frac{1}{2} \left(x_i + \frac{20}{x_i} \right) \quad (5 \text{ marks})$$

QUESTION TWO (30 MARKS)

(a) Find the real root of the equation $x^3 + x - 1 = 0$ using the iterative formula

$$x_{i+1} = (1 - x_i)^{1/3} \quad (5 \text{ marks})$$

(b) Determine the value of y_{-1} if $y_0 = 2$, $y_1 = 9$, $y_2 = 28$, $y_3 = 65$, $y_4 = 126$ and $y_5 = 217$.

(5 marks)

(c) By use of stirlings formula and the data below, determine the value of $y(1.22)$.

x:	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y:	0.84147	0.89121	0.93204	0.96356	0.98545	0.99749	0.99957

x: 1.7 1.8

y: 0.99385 0.97385 (10 marks)

QUESTION THREE (20 MARKS)

- (a) Use the Newton-Raphson method to find the root of the equation $x^3 - 2x - 5 = 0$
Correct to 5 decimal places between 2 and 3. (5 marks)
- (b) Compute the definite integral $\int_0^1 \frac{dx}{1+x}$ by use of Simpson one third rule taking $n = 6$
(5 marks)
- (c) Use regular falsi method to find a real root of the function: $x \log_{10} x - 1.2 = 0$.
(10 marks)

QUESTION FOUR (20 MARKS)

- (a) Show that $\nabla^3 y_k = y_k - 3y_{k-1} + 3y_{k-2} - y_{k-3}$ (3 marks)
- (b) Construct a backward difference table from the following data:
 $\sin 30^\circ = 0.5000$, $\sin 35^\circ = 0.5736$, $\sin 40^\circ = 0.6428$, $\sin 45^\circ = 0.7071$.
Assuming that the third backward difference is a constant, determine the value of
 $\sin 25^\circ$. (7 marks)
- (c) $y = f(x)$ is a polynomial of degree 3 and the following table gives the values of x and y .
locate and correct the wrong values of y .
- | | | | | | | | |
|----|---|----|----|----|-----|-----|-----|
| x: | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| y: | 4 | 10 | 30 | 75 | 160 | 294 | 490 |
- (10 marks)

QUESTION FIVE (20 MARKS)

- (a) Calculate the first and second derivatives of the function tabulated below at the point $x = 1.2$
- | | | | | | | | |
|----|--------|--------|--------|--------|--------|--------|--------|
| x: | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 2.2 |
| y: | 2.7183 | 3.3201 | 4.0552 | 4.9530 | 6.0496 | 7.3891 | 9.0250 |
- (10 marks)
- (b) Determine the value of $\sqrt[3]{24}$ correct to 3 d.p using Newton-Raphson method. (6 marks)
- (c) Find the divided difference of $f(x) = x^3 + x + 2$ for the inputs
1, 3, 6, 11.
(4 marks)