KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/2010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF COMPUTER SCIENCE

COURSE CODE: **MATH 314**

COURSE TITLE: **NUMERICAL ANALYSIS**

STREAM: **Y3S1**

DAY: **FRIDAY**

TIME: **2.00 – 4.00 P.M.**

DATE: **19/03/2010**

INSTRUCTIONS:

Answer Question ONE and any other TWO Questions.

PLEASE TURN OVER

QUESTION ONE: (30 MARKS)

a). Convert the binary number 1110011.1101001 into

i). Octal number

ii). Hexadecimal Number

iii). Decimal Number [6 marks]

b). Consider the system of linear equations

$$x_1 - 2x_2 + 10x_3 = 35$$

$$10x_1 - 2x_2 + x_3 = 17$$

$$-x_1 + 5x_2 - x_3 = -14$$

i). Write down the general Gauss-Seidel iteration for this system.

ii). Starting with $x_1^{(0)} = 0.8$, $x_2^{(0)} = -1.5$ and $x_3^{(0)} = 2.5$, solve the system correct to 5 decimal places using the Gauss-Seidel iteration [8 marks]

c). Determine the characteristic equation of the matrix

$$A = \begin{pmatrix} 8 & 3 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & -8 \end{pmatrix}$$

Hence determine the largest positive eigenvalue of A correct to 7 decimal places. [7 marks]

d). Consider the nonlinear equation

$$f(x) = x^2 - xe^x + 1 = 0$$

show that the equation has a root in the interval [0 , 1]. Hence carry out three iterations of the bisection method to determine the root, giving your answer correct to 4 decimal places.

[5 marks]

e). Obtain the Taylor series expansion of $f(x+\beta)$ about β up to the term of order β^5 [5 marks]

QUESTION TWO: (20 MARKS)

a). Use Aitkens' method to find the roots of

$$f(x) = x^3 - 13x + 8 = 0$$

on the interval I = [0 , 1] using the iteration scheme $x_{n+1} = \frac{x^3{}_n + 8}{13}$ correct to 9 decimal places

[8 marks]

b). Show that the convergence of Newton Raphson method is of order 2 [6 marks]

c). Find the inverse of the matrix $B = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ [8 marks]

QUESTION THREE: (20 MARKS)

a). Find the roots of $f(x) = x^3 + 5x - e^x + 2 = 0$ using Newton –Raphson method with $x_0 = 5.0508$

[6 marks]

b). Consider the system

$$x_1 + 2x_2 + 40x_3 = 48.0035$$

$$20x_1 + 2x_2 + x_3 = 65.9901$$

$$2x_1 + 20x_2 + x_3 = 56.9991$$

i). Solve the system using Gaussian elimination method correct to 6 significant places [8 marks]

ii). Using the results in (i) above, carry out iterative improvements so that the solution is correct to 7 significant figures. [6 marks]

QUESTION FOUR: (20 MARKS)

a). Stating your assumptions, derive the Jacobi iteration for solving the linear system, $A\breve{x} = \breve{b}$ where A is a square matrix of order n. [9 marks]

b). consider the linear system

$$x_1 + 5x_2 + x_3 = 14$$

$$x_2 + 5x_3 = 17$$

$$5x_2 + x_3 = 7$$

a). Starting with the initial approximations $\tilde{x}^0 = [1.0018,\ 2.0039\ \ 2.9996]^T$ and using the Jacobi iteration, determine the solutions correct to 4 decimal places. [11 marks]

QUESTION FIVE: (20 MARKS)

a). The following table of values is given for x and for f(x) as shown below.

x	-1	1	2	3	4	5	7
f(x)	1	1	16	81	256	625	2401

Using the formula $f'(x_1) = \frac{[f(x_2) - f(x_0)]}{2h}$ and the Richardson extrapolation, find $f'(3)$ [7 marks]

b). Find the Jacobian matrix of the equations

$$f_{1(x,y)} = x^2 + y^2 - x = 0$$

$$f_{2(x,y)} = x^2 - y^2 - y = 0$$

At the point (1, 1) using the method

$$\left(\frac{\partial y}{\partial x}\right)_{x_i, y_j} = \frac{f_{i+1,\ j} - f_{i-1,\ j}}{2h}$$

$$\left(\frac{\partial y}{\partial y}\right)_{x_i, y_j} = \frac{f_{i,\ j+1} - f_{i,\ j-1}}{2h}$$

With $h = k = 1$ [6 marks]

c). Evaluate the integral

$$I = \int_1^2 \int_1^2 \frac{dxdy}{x + y}$$

Using the Trapezoidal Rule with h = k = 0.5 [7 marks]