

KABARAK



UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

**FOR THE DEGREE OF BACHELOR OF EDUCATION
SCIENCE**

COURSE CODE: MATH 314

COURSE TITLE: NUMERICAL ANALYSIS 1

STREAM: SESSION VI & VII

DAY: MONDAY

TIME: 2.00 – 4.00 P.M.

DATE: 06/04/2009

INSTRUCTIONS:

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

- a. The following table gives the values of y which a polynomial of degree 5. It is known that $y = f(x)$ has an error. Correct the error.

| | | | | | | | | |
|----|---|---|----|-----|------|------|------|-----------|
| x: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | |
| y: | 1 | 2 | 33 | 254 | 1025 | 3126 | 7777 | (4 marks) |

- b. In a triangle ABC. $A=6$ cm, $c=15$ cm, $\angle B = 90^\circ$. Find the possible error in the computed value of B , if the errors in the measurements of a and c are 1mm and 2mm respectively. (5 marks)
- c. Prove that: $\Delta [f(x)g(x)] = f(x+h)\Delta g(x) + g(x)\Delta f(x)$ (4 marks)
- d. Use Newton's Backward formula to find $y(-1)$ if $y(0) = 2$, $y(1) = 9$, $y(2) = 28$, $y(3) = 65$, $y(4) = 126$, $y(5) = 217$. (5 marks)

- e. Find the value of $[\frac{\Delta^2}{E}]e^x \cdot \frac{Ee^x}{\Delta^2 e^x}$. Take $h=1$ (4 marks)

- f. Proof the following:

(i) $\nabla\Delta = \Delta - \nabla = \delta^2$ (4 marks)

- g. Given the following function values, find a second degree Lagrange interpolation.

$x_0 = -2, y_0 = -8.52, x_1 = 1, y_1 = 3.00, x_2 = 2, y_2 = 7.48$ (4 marks)

QUESTION TWO (20 MARKS)

- a. Evaluate the following:

(i) $(\frac{\Delta^2}{E})x^3$ (4 marks)

(ii) $(E^{-1}\Delta)x^3$ (3 marks)

(iii) $(\nabla + \Delta)^2 (x^2 + x + 1)$ (5 marks)

(iv) If $x_0 = 3, x_1 = 12, x_2 = 81, x_3 = 200, x_4 = 100, x_5 = 8$, find the value of $\Delta^5 x_0$ (3 marks)

b. Find the cubic polynomial from the data below:

| | | | | | |
|----|----|-----|----|---|----|
| x: | 0 | 1 | 2 | 3 | 4 |
| y: | -5 | -10 | -9 | 4 | 35 |

(5 marks)

QUESTION THREE (20 MARKS)

a. The following table gives the population of a certain country. Find the population in the year 2001 using the Advancing difference formula:

| | | | | |
|-------------------------|------|------|------|------|
| Year | 1941 | 1951 | 1961 | 1971 |
| Population(in millions) | 352 | 405 | 473 | 554 |

(5 marks)

b. Use the table below to find $f(0.36)$ using Bessel's formula.

| | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|
| x: | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
| f(x): | 1.172 | 1.008 | 0.878 | 0.782 | 0.720 | 0.692 |

(5 marks)

c. Given the following tabulated function:

| | | | | |
|-------|-----|-------|-------|------|
| x: | 1.0 | 2.0 | 3.0 | 4.0 |
| f(x): | 150 | 36.75 | 17.33 | 9.19 |

Use Newton's Backward interpolation formula to evaluate $f(5.0)$ (5 marks)

d. Find the missing term:

| | | | | | | |
|----|----|----|----|-----|-----|-----|
| x: | 7 | 9 | 11 | 13 | 15 | 17 |
| y: | 32 | 78 | - | 144 | 257 | 381 |

(5 marks)

QUESTION 4 (20 MARKS)

a. Use Stirling's formula to calculate $f^{-1}(2)$ from the table below:

x: 1.7 1.8 1.9 2.0 2.1 2.2 2.3

f(x): 0.58824 0.55556 0.52632 0.50000 0.47619 0.45455 0.41667

(6 marks)

b. From the following table, calculate:

(1) $\int_0^1 f(x)dx$ (2) $\int_0^2 f(x)dx$ (3) $\int_2^3 f(x)dx$

x: 0 1 2 3 4 5 6 7 8 9 10

f(x): 1.0 0.9514 0.9182 0.8975 0.8873 0.8862 0.8935 0.9086 0.9314 0.9618 1.0

(9 marks)

c. From the following data, estimate $\int_0^2 y(x)dx$ using Simpson's three – eighth rule

x: 0 .25 .50 .75 1.00 1.25 1.50 1.75 2

y: 1.000 1.284 1.649 2.117 2.718 3.490 4.482 5.755 7.389

(5 marks)

QUESTION 5 (20 MARKS)

a. Assuming that a root of $f(x) = x^3 - 9x + 1$ lies in the interval (2, 4), find that root by Newton Raphson method. (5 marks)

b. Solve $e^x - 3x = 0$ by iterative method (5 marks)

c. Find a real root of the equation $\cos x = 3x - 1$ correct to 4 decimal places by iteration method (5 marks)

e. Find f(x) from the table below. Also find f (7). (5 marks)

x: 0 1 2 3 4 5 6

f(x): -1 3 19 53 111 199 323