

# UNIVERSITY EXAMINATIONS 

 2010/2011 ACADEMIC YEAR FOR THE DEGREE OF BACHELOR OF COMPUTER SCIENCE
## COURSE CODE: MATH 314

COURSE TITLE: NUMERICAL ANALYSIS
STREAM:
DAY:
FRIDAY
TIME:
9.00-11.00 A.M.

DATE:
18/03/2011

## INSTRUCTIONS:

Answer question ONE and any other TWO questions

1. a) Let $x=3.55 \pm 0.05$ and $y=2.73 \pm 0.05$ determine the maximum error in calculating

$$
\begin{equation*}
f(x)=\frac{x}{y} . \tag{5marks}
\end{equation*}
$$

b) Two quantities are found to be related as below.

| $\mathrm{X}:$ | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{~F}(\mathrm{X})$ | 3.1 | 4.0 | 4.2 | 3.8 | 2.9 | 2.8 | 2.7 |

Assuming $\mathrm{f}(\mathrm{x})$ is continous find $\int_{1}^{4} f(x) d x$ numerically using both trapezoidal and
Simpson's rules.
c) Use Lagrange interpolation polynomial to calculate $f(2)$ from the following table

| $\mathrm{x}:$ | 0 | 1 | 3 |
| :--- | :--- | :--- | :--- |
| $\mathrm{f}(\mathrm{x})$ | 1 | 3 | 35 |

d) Use the Newton-Raphson's iteration to estimate the square root of 2 starting with $x_{0}=1.4$ upto 3 iterations correct 5 decimal places.
e) Find $f(x)$ from the following table and also $f(7)$.

| $\mathrm{x}:$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{~F}(\mathrm{x}):-$ | 1 | 3 | 19 | 53 | 111 | 199 | 323 |

2. a) Consider the sequence of values of $f(x)=(0,0,0, \varepsilon, 0,0,0)$ where $\varepsilon$ is an error. Show that
(i) the error spreads and increase in magnitude as the order of the difference is increased.
(ii) the error in each column have binomial coefficients.
b) Find and correct a single error in the following table

| $\mathrm{x}:$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{y}:$ | 0 | 0 | 1 | 6 | 24 | 60 | 120 | 210 | (7 marks) |

c) Perform the computation $0.0218 \times 179$
(i) Exactly
(ii) using three truncating arithmetic
(iii) Using three digit rounding arithmetic.

Comment on the errors generated by (i) and (ii).
3. a) Find the divided differences of $f(x)=x^{3}+x+2$ for the arguments $1,3,6$, 11. (4 marks)
b) Using the Newton's divided formula find $\mathrm{f}(\mathrm{x})$ and $\mathrm{f}=(6)$ from the values below.

| $\mathrm{x}:$ | 1 | 2 | 7 | 8 |
| ---: | ---: | ---: | ---: | ---: |
| $\mathrm{f}(\mathrm{x}):$ | 1 | 5 | 5 | 4 |

c) Apply Newton-Raphson's method, find correct to four decimals the root between 0 and 1 of the equation $x^{3}-6 x+4=0$ Take $x_{0}=0.7$
4. a) The population increase of a certain town is given below. Find the rate of growth of the population in 1931 and 1971.
$\begin{array}{llllll}\text { Year, x: } & 1931 & 1941 & 1951 & 1961 & 1971\end{array}$
Pop in thousands,y: $\begin{array}{lllllll}40.62 & 60.80 & 79.95 & 104.56 & 132.65 & \text { (10 marks) }\end{array}$
b) By Stirling and Bessel method compare the interpolation at $\mathrm{x}=0.35$ from the data

| $\mathrm{x}:$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{f}(\mathrm{x}):$ | 1.40 | 1.56 | 1.76 | 2.00 | 2.28 | 3.18 | (10 marks) |

5. a) Using $\sin (0.1)=0.09983$ and $\sin (0.2)=0.19867$ find an approximate value of $\sin (0.15)$ by lagrange interpolation.
(7 marks)
b) Evaluate $I=\int_{0}^{2} \frac{x}{1+x^{2}} d x$ taking a subdivision of 0.25 using
(i) Trapezoidal rule
(ii) Simpsos's rule
(iii) Direct Integration.
(13 marks)
