

KABARAK



UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF COMPUTER SCIENCE
AND BACHELOR OF ECONOMICS & MATHEMATICS

COURSE CODE: MATH 314

COURSE TITLE: **NUMERICAL ANALYSIS EXAM**

STREAM: Y3S1

DAY: MONDAY

TIME: 11.00 – 1.00 P.M.

DATE: 08/12/2008

INSTRUCTIONS TO CANDIDATES:

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

PLEASE TURN OVER

QUESTION ONE(30 MARKS)

(a). In a triangle ABC, $a = 5$ cm, $c = 15$ cm, $\angle B = 90^\circ$. Find the possible error in the computed value of A if the errors in a and b are $\frac{1}{2}\%$ and $\frac{1}{3}\%$ respectively. (6 marks)

(b). Locate the error in the following entries and correct it: 125 132.651

140.608 148.877 157.464 166.357 175.616 185.193 195.112

205.379 216. (5 marks)

(c). Use the Advancing difference formula to find $f(50)$:

x:	15	20	25	30	35
f(x):	1558	1806	2094	2427	2814

(5 marks)

(e). Evaluate: $(\Delta + \nabla)^2(x^2 + x + 1)$, $h = 1$. (5 marks)

(f). Use Newton – Raphson method to find $\sqrt{3}$ to six decimal places with

$x_0 = 2$. (5 marks)

(g). Estimate the missing term in the following:

x:	1	2	3	4	5	6	7
y:	2	4	8	_	32	64	128

(4 marks)

QUESTION 2 (20 MARKS)

(a). Use the iterative method to find the real root of the equation $3x - \log_{10} x = 6$ if the root lies in the interval (2, 3). (5 marks)

(b). Solve the equation $x^4 - x - 9 = 0$ by Newton Raphson Method. (5 marks)

(c). Show that the iterative formula for finding the reciprocal of n is

$$x_{i+1} = x_i(2 - nx_i) \text{ and hence find the value of } \frac{1}{31}. \quad (5 \text{ marks})$$

(d). Solve the following equation by Regula Falsi method: $x^3 + x - 1 = 0$. (5 marks)

QUESTION 3 (20 MARKS)

a. Use Bessel's formula to find $f(0.36)$ from the following table:

x:	0.1	0.2	0.3	0.4	0.5	0.6
y:	1.172	1.008	0.878	0.782	0.720	0.692

$$\left[y = \frac{y_0 + y_1}{2} + v\Delta y_0 + \left(\frac{v^2 - \frac{1}{4}}{2}\right) \frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2} + \dots \right]$$

(5 marks)

b. Using the table below, evaluate $f^{(11)}(4)$ using Stirling's formula:

$$y = y_0 + u \frac{\Delta y_{-1} + \Delta y_0}{2} + \frac{u^2}{2} \Delta^2 y_{-1} + \frac{u(u^2 - 1)}{3!} \frac{\Delta^3 y_{-2} + \Delta^3 y_{-1}}{2} + \frac{u^2(u^2 - 1)}{4!} \Delta^4 y_{-2} + \dots$$

x:	2.4	2.8	3.2	3.6	4.0	4.4	4.8
f(x):	0.18322	0.34164	0.48324	0.61245	0.73205	0.84391	0.94936

(5 marks)

(c). Given the following tabulated function:

x:	1.0	2.0	3.0	4.0
f(x)	150	36.75	17.33	9.19

Find $f(5.0)$. (5 marks)

(d). Use the Trapezoidal rule to find $\int_1^2 \frac{1}{1+x} dx$, $h = 0.1$. Compare with exact

solution. (5 marks)

QUESTION 4 (20 MARKS)

(a). Calculate a fourth divided difference for the following values:

x:	0	1	2	4	5
y:	0	16	48	88	0

(4 marks)

(b). Use Lagrange's interpolation formula to find $f(4.3)$ for the following:

x:	0	1.0	2.0	3.8
f(x):	0	0.569	0.791	0.224

(5 marks)

(c). Find the polynomial of degree three which takes the values prescribed below using Lagrange's method

x	0	1	2	4
y	1	1	2	5

(6 marks)

(d). Evaluate: $(2\Delta + 3)(E + 2)(3x^2 + 2)$ where $h = 1$ (5 marks)

QUESTION 5 (20 MARKS)

(a). Prepare a forward difference table for the following function:

x	1	2	3	4	5
f(x)	6	10	46	138	430

assuming the function is a polynomial, interpolate for $f(4.31)$ using forward difference interpolation with $x=4$ as starting point.

(5 marks)

(b). Given the following function values:

x	0	0.5	1.0	1.5	2.0	2.5
f(x)	2.014	3.221	4.701	7.710	13.594	23.580

Find $f(3.0)$.

(5 marks)

(c). Prove:

(i) $E^{\frac{1}{2}} = \mu + \frac{1}{2}\delta$ and $E^{-\frac{1}{2}} = \mu - \frac{1}{2}\delta$

(ii) $\mu\delta = \frac{1}{2}\Delta E^{-1} + \frac{1}{2}\Delta$

(iii) $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{1}{4}\delta^2}$

(10 marks)