KENYA METHODIST UNIVERSITY

END OF 3RD TRIMESTER 2008 EXAMINATIONS

FACULTY	:	ARTS AND SCIENCES
DEPARTMENT	:	COMPUTER INFORMATION SYSTEMS
UNIT CODE	:	MATH 103
UNIT TITLE	:	CALCULUS 1
TIME	:	2 HOURS

Instructions:

- *Answer question 1 and any other 2 questions.*

Question 1 (30 marks)

a) Find the limit of the following functions:

i) $\lim\limits_{x \to 1} \frac{x+2}{\sqrt{x+3-1}}$ (2 mks)

ii) $\lim\limits_{x \to 1} \frac{x^3-1}{x-1}$ (2 mks)

b) Find the derivative of the indicated functions using the method of differentiation from first principles:

i) $y = \frac{1}{x}$ (3 mks)

ii) $y = 2x^2 + 3x+1$ (3 mks)

c) Given the function $y = x^2 + x + 1$. Find the increment in the argument and the increment in the function if the argument x changed its value from $x_1 = 2$ to $x_2 = 2.5$

d) Compute the indicated integrals:

i) $\int_{1}^{\frac{1}{3}} \frac{dx}{(4-3x)^2}$ (3 mks)

ii) $\int \sqrt{(3x+2)^2}\,dx$ (4 mks)

e) i) Find the area enclosed by the graphs of the functions:
$f(x)=2x^2$ and $g(x) = 2x + 4$ (4 mks)

ii) Find the equations of the tangent and normal to the curve $x^3+x^2y+y^3-7 = 0$ at the point $x=2$, $y=3$

(6 mks)

Question 2 (20 marks)

a) The parametric equations of a curve are $x = \frac{3t}{1+t}$, $y = \frac{t^2}{1+t}$. find the equations of the tangent and normal at the point for which t=2. (7 mks)

b) Sketch the curve whose equation is:

$y = \frac{(x+2)(x-3)}{x+1}$ (12 mks)

c) Show that: $y = a\cos\left(\frac{x}{a}\right)$ satisfies the differential equation $\frac{d^2y}{dx^2} = \frac{w}{H}\sqrt{1-\left(\frac{dy}{dx}\right)^2}$

provided that $a=\frac{H}{w}$, where H and w are constants. (8 mks)

Question 3 (20 marks)

a) Find two positive numbers whose sum is 20 and whose product is as large as possible. (6 mks)

b) Test the continuity of the given function at the given points.

$Y = \frac{x^2-4}{(x-2)(x-3)}$, at

i) x=3 (3 mks)

ii) x=2 (3 mks)

iii) Which discontinuity, if any, is removable? (2 mks)

c) If y = cos2t and x = sint, find the equations of the tangent and normal to the curve at $t = \frac{\pi}{6}$ (6 mks)

Question 4 (20 marks)

a) Find the position of S(t) of a particle moving on a line if $\frac{ds}{dt} = v = 5\cos\pi t\ m/s$ and S(0)=2. Also, find the total distance travelled by the particle from t=0 to $t=\frac{3}{2}$ seconds, and the particles' displacement for this time period. (10 mks)

b) Find the derivative of:

i) $f(x) = \frac{x^2\sqrt{5x+1}}{(3x-2)^3}$ (5 mks)

ii) $e^{xy} = x^2+y$ (5 mks)

Question 5 (20 marks)

a) Find the area generated when the arc of the parabola $y^2=8x$ between x=0 and x=2, rotates about the x-axis. (8 mks)

b) Find the limit of the following functions:

i) $\lim_{x\to 2}\left[\dfrac{2x^2-4x}{x-2}\right]$ (4 mks)

ii) $\lim_{x\to -\infty}\left[\dfrac{4x^5-3x^3+10}{2x^5+\frac{1}{2}x^4-10}\right]$ (4 mks)

c) Answer the following, giving reasons:

i) If a function is continuous everywhere, does it mean that it is differentiable everywhere? (2 mks)

ii) If a function is differentiable everywhere, does it mean that it is continuous everywhere? (2 mks)