KENYA METHODIST UNIVERSITY

END OF 3RD TRIMESTER 2008 EXAMINATIONS

FACULTY	:	ARTS AND SCIENCES
DEPARTMENT	:	COMPUTER INFORMATION SYSTEMS
UNIT CODE	:	MATH 211
UNIT TITLE	:	DISCRETE STRUCTURES
TIME	:	2 HOURS

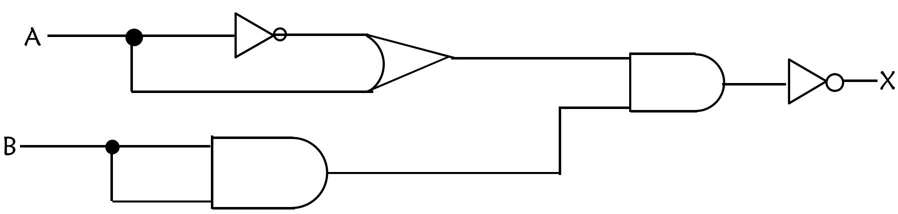
Instructions:
- *Answer question ONE and any other TWO questions.*

Question 1 (30 marks)

a) i) Verify that the proposition
$P\Lambda(q\Lambda \sim P)$ is a contradiction (2 mks)

ii) Show that $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$, $n \geq 1$
by mathematical induction. (4 mks)

b) Find the output of the following network as shown below and design a simple network having the same output.



(5 mks)

c) A class has 12 boys and four girls. Suppose three students are selected at random from the class. Find the probability that they are all boys. (4 mks)

d) i) How many different signals, each consisting of eight flags hung in a vertical line, can be formed from a set of four indistinguishable red flags, three indistinguishable white flags and a blue flag? (3 mks)

ii) A farmer buys 3 cows, 2 pigs and 4 hens from a man who had 6 cows, 5 pigs and 8 hens. How many choices does the farmer have? (3 mks)

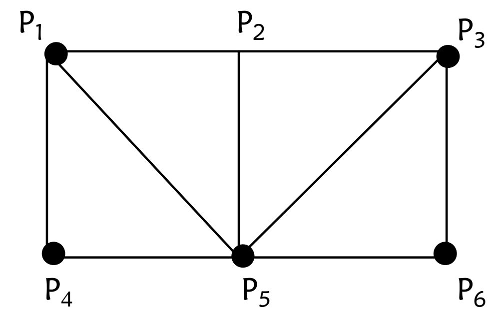
e) In the graph below, determine whether the following are paths, simple paths, trails, circuits or simple circuits. (4 mks)

i) (P_4, P_1, P_2, P_5, P_1, P_2, P_3, P_6)

ii) (P_4, P_1, P_5, P_2, P_6)

iii) (P_4, P_1, P_5, P_2, P_3, P_5, P_6)

iv) $(P_4, P_1, P_5, P_3, P_6)$



f) Consider the following statement:

if x>3 then y:=x + 5, else do x:=x-1

y: = 8.x end do

what is the value of y after execution of this statement for the following values of x:

i) x = 4 ii) x = 2 (3;2 mks)

Question 2 (20 marks)

a) i) Write the binary search algorithm. (5 mks)

ii) Hence or otherwise, show the sequence of steps in using a binary search to find the number 3 in the list 1, 2, 3, 5, 7, 9, 11, 13, 15 how many times is 3 compared with an element in the list? (6 mks)

b) A student must take five classes from three areas of study. Numerous classes are offered in each discipline, but the student cannot take more than two classes in any given area.

i) Using the pigeonhole principle, show that the student will take at least two classes in one area. (4 mks)

ii) Using the inclusion-exclusion prinicple, show that the student will have to take at least one class in each area. (5 mks)

Question 3(20 marks)

a) Show that ~r is a valid conclusion from the premises $p \Rightarrow \sim q$, $r \Rightarrow p$, q

i) with truth table (5 mks)

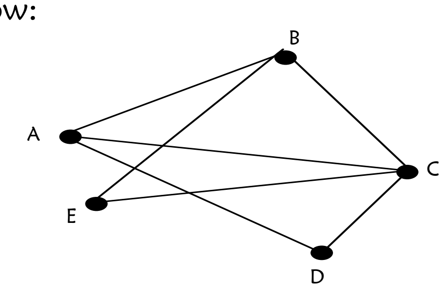
ii) without truth table (5 mks)

b) i) In how many ways can 10 students be divided into three teams, one containing four students and the others three. (4 mks)

ii) Consider a tournament in which each of n players plays against every other player and each player wins at least once. Show that there are at least two players having the same number of wins. (6 mks)

Question 4 (20 marks)

a) Consider the figure below:



i) Describe formally the graph G in the diagram. (4 mks)

ii) Find the degree of each vertex and verify that the sum of the degrees of the vertices of a graph G is equal to twice the number of edges in G. (6 mks)

b) Use mathematical induction to prove that:

i) $2n<n!$ for every positive integer n with $n\geq 4$ (5 mks)

ii) $\sum_{j=0}^{n} ar^{j} = a+ar+ar^{2}+\ldots+ar^{n} = \dfrac{ar^{n+1}-a}{r-1}$ when $r\neq 1$ (5 mks)

Question 5 (20 marks)

a) i) For any two events A and B, show that $P(A\cup B) = P(A) + P(B) - P(A\cap B)$ (6 mks)

ii) The probability that A hits a target is $\frac{1}{4}$, and the probability that B hits the target is $\frac{2}{5}$ Both shoot at the target. Find the probability that at least one of them hits the target. (4 mks)

b) Determine whether the game defined by the matrix below is strictly determined.

$$\begin{bmatrix} 3 & 0 & -2 & -1 \\ 2 & -3 & 0 & -1 \\ 4 & 2 & 1 & 0 \end{bmatrix}$$ (6 mks)

c) Define the following terms:

i) two-person game (2 mks)

ii) strictly determined game (2 mks)