KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **MATH 312**

COURSE TITLE: ORDINARY DIFFERENTIAL EQUATION I

STREAM: **SESSION V & VII**

DAY: **THURSDAY**

TIME: **2.00 – 4.00 P.M.**

DATE: **27/11/2008**

INSTRUCTIONS:

Answer Question ONE and any other TWO

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) Find the general solution of the following differential equation.

i. $\frac{dy}{dx} = 3x^2$ **(2 marks)**

ii. $\frac{xdy}{dx} = \tan y$ **(3 marks)**

(b) Test whether the following differential equation are exact and hence solve them :

i. $2xy\frac{dy}{dx} + y^2 = e^{2x}$ **(4 marks)**

ii. $(x+2y)dx + (2x+y)dy = 0$ **(4 marks)**

(c) Use the method of undetermined coefficient to find P.I of

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 2x - 40\cos 2x$$

(4 marks)

(d) Solve the differential equation:

i. $\frac{d^2y}{dx^2} + \frac{3dy}{dx} + 2y = 0$

(3marks)

ii. $\frac{d}{dx}(2y-4) = 3x + 4x - 4$

Given that $y = 3$, $x = 1$ **(3 marks)**

(e) The population of a country increases at a rate proportional to the current population. If the population doubles in 40 years, in how many years will it triple?

(5 marks)

QUESTION TWO (20 MARKS)

(a) Use the variation of parameters to solve the following differential equation completely.

$$\frac{dy}{dx} + y = \sec x \tan x$$

(b) Find the integrating factor of the differential equation;

$$\frac{dy}{dx} + y\cot x = \cos x$$

And hence solve it. **(5 marks)**

(c) Solve completely the following differential equation

$$\frac{d^2y}{dx^2} - 9\frac{dy}{dx} + 20y = 20x^2$$ ` **(5 marks)**

(d) By making the substitution $\frac{dy}{dx} = p$,

Solve the differential equation:

$$(1 + x^2)\frac{dy}{dx} = 2x\frac{dy}{dx}$$ **(3 marks)**

QUESTION THREE (20 MARKS)

(a) Determine the particular solution of

$(D^2 -2D +1)y = 0$ given that x = 0 ,y = 0 and that x =1 when y = e **(5 marks)**

(b) Determine the P.I of

$(D^3 +4D)y = \sin 2x$ using the inverse operator method. **(3 marks)**

(c) Use the method of undetermined coefficient to find the P.I of

$(D^2 + 4D + 5)y = 10x^2 + x$

What will be the solution of the above differential equation? **(3 marks)**

(d) Show that;

$$\frac{1}{D-a}x = e^{ax}\int xe^{-ax}\,dx$$

Where $D = \frac{d}{dx}$ **(5 marks)**

QUESTION FOUR (20 MARKS):

(a) Show that equation $y = Ae^{2x} + Be^{-x}$ is the solution of the differential equation define by;

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$ **(5marks)**

(b) Solve the differential equation:

$$\frac{d^2 y}{dx^2} + \frac{11dy}{dx} + 24y = 0$$

Subject to y = 0, When x = 0 and $\frac{dy}{dx} = 7$ when x = 0 **(5 marks)**

(c) Determine the particular integral for the non homogenous equations

$$\frac{d^2y}{dx^2} - \frac{4dy}{dx} - 12y = 3e^{5t}$$

Using the method of undetermined coefficient **(4marks)**

(d) Find the general solution of the differential equation:

$$\frac{d^2y}{dx^2} - \frac{6dy}{dx} - 2y = 0$$ **(6 marks)**

QUESTION FIVE (20 MARKS)

a) The population of a city increases at a rate which is proportional to its present population .If the initial population was 500,000, find the expression of the population of the city at time t **(5 marks)**

b) If in 30 years the population of the city in (a) above increases by 100,000, what will be the population of the city after 150 years. **(5 marks)**

c) Uranium disintegrates at a rate proportional to the amount present at any instant. If $m_1$and m_2 grammes of uranium are present at times T_1 and T_2 respectively, show that half –life of uranium is given by

d) $T_{½} = \frac{T_2 - T_1}{Log\left(\frac{m_1}{m_2}\right)}\log_2$ **(5 marks)**

e) Solve the equation

$(D^2 +1)\ y = e^{2x}$ **(5 marks**