

KABARAK



UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

**FOR THE DEGREE OF BACHELOR OF EDUCATION
SCIENCE**

COURSE CODE: MATH 312

COURSE TITLE: ORDINARY DIFFERENTIAL EQUATIONS

STREAM: SESSION V

DAY: TUESDAY

TIME: 2.00 – 4.00 P.M.

DATE: 07/04/2009

INSTRUCTIONS:

Answer question **ONE** and any other **TWO** questions

PLEASE TURN OVER

Question One (30 Marks)

- a) Given the differential equation $y'' - 2y' + 2y = 0$ determine whether $y = (c_1 \cos x + c_2 \sin x)$ is its complete primitive where c_1 and c_2 are arbitrary constants. (4 Marks)
- b) Determine the values of c_1 and c_2 so that $y(x) = c_1 e^x + c_2 e^{-x} + 2 \sin x$ will satisfy the conditions $y(0) = 0$ and $y'(0) = 0$. (5 Marks)
- c) Solve $(x - 4) \frac{dy}{dx} = y$. (6 Marks)
- d) Determine if the functions $y_1(x) = \cos x$ and $y_2(x) = \sin x$ are linearly dependent or linearly independent. (3 Marks)
- e) Solve differential equation $y'' = y$. (6 Marks)
- f) The differential equation for a circuit in which self-inductance and capacitance neutralize each other is $L \frac{d^2i}{dt^2} + Ri = 0$. Find the current i as a function of t given that i is the maximum current and $i = 0$ when $t = 0$. (6 Marks)

Question Two (20 Marks)

- a) Solve the following homogeneous linear differential equation $y'' + 3y' + 2y = 0$. (8 Marks)
- b) Solve the simultaneous equations,
 $y_1 + y_2 = 0 \dots \dots \dots (1)$
 $y_1 - y_2 = 0 \dots \dots \dots (2)$ (6 Marks)
- c) Solve the initial value problem: $y' = \sqrt{y+1}$, $y(0) = 1$. (6 Marks)

Question Three (20 Marks)

- a) Show that the differential equation $(3x^2 + 2y) dx + (2xy - 3x^2) dy = 0$ is not exact. Then reduce it to an exact form and solve it. (12 Marks)
- b) Given the differential equation $(x + y) \frac{dy}{dx} = 2 \cos x + 3x^2 + 2 + 3$. Determine its complementary function and the particular integral, hence write the general solution. (8 Marks)

Question Four (20 Marks)

- a) The mass of a crystalline deposit increases at a rate which is proportional to its mass at that time. The deposit has started around a crystal seed of 5 .
- (i) Find an expression of its mass at time . (4 Marks)
- (ii) If in 30 minutes the mass of the deposit increases by 1 , what will be the mass of the deposit after 10 hours. (4 Marks)
- b) Solve the differential equation $\frac{dy}{dx} - 5y + 6 = 0$ subject to $y(0) = -1$ and $y(1) = 1$. (9 Marks)
- c) Use the method of integrating factor to solve the differential equation: $\frac{dy}{dx} + y = -3$. (3 Marks)

Question Five (20 Marks)

- a) An electric circuit consists of an inductance of 0.1 H, a resistance of 20 Ω and a condenser of capacitance 25 μF .
- (i) Write down the differential equation for the circuit. (3 Marks)
- (ii) Find the Charge, q , and the current, i , at any time t given that at $t = 0$, $q = 0.05$ C, $i = 0$. (12 Marks)
- b) Use the method of separation of variables to solve the differential equation $\frac{dy}{dx} = (1 + y)^2$. (5 Marks)